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Two Degree-Of-Freedom Online Compensation of a Piezoelectric Micro-Positioning Unit

Christopher Schindlbeck[1,*], **Christian Pape**[1,**], and **Eduard Reithmeier**[1,***]

[1] Institute of Measurement and Automatic Control, Leibniz Universität Hannover, Nienburger Str. 17, 30167 Hannover, Germany

Due to their almost unlimited resolution and fast dynamics, piezoelectric actuators are a common choice for mechatronic systems targeting positioning tasks with high demands on precision. However, these piezoelectric actuators inherently suffer from nonlinear characteristics (mainly hysteresis and creep effects) which need to be addressed by appropriate control strategies. The operator-based modified Prandtl-Ishlinksii (mPI) approach does not only model hysteresis effects with asymmetries and creep effects but also provides an analytical solution for its inverse model. Online feedforward compensation of the aforementioned nonlinear effects can be realized by using the inverse model and additional weight adaptation. In this paper, online compensation via the mPI model is applied to a commercial micro-positioning unit driven by piezoelectric actuators with more than one degree of freedom (DOF). For validation of the proposed approach, two coupled trajectories in the X-Y plane are utilized. Subsequent tracking error analysis validates the efficacy of the stated approach.

1 Introduction

It is well known that open-loop control of piezoelectric actuators exhibits nonlinearities, predominantly hysteresis and creep effects which need to be addressed appropriately in order to increase performance. For hysteresis modeling, model-based approaches resort to a variety of models which either mimic physical properties (physics-based model) or are independent of the underlying physical model (phenomenological models). The latter can be subdivided into differential-equation-based models, operator-based models, or other models such as neuro/fuzzy or ellipse-based models [1]. The operator-based mPI model [2] offers the advantage that hysteresis with asymmetric behavior and creep can be modeled in a unified way. Furthermore, the model can be inverted analytically which is beneficial for online compensation. Therein, the inverse model is typically applied as feedforward compensator of piezoelectric nonlinearities. A recursive online approach is presented in [3] yielding robustness against varying final times, ranges of motions, and stationary states. Although online capabilities are provided by the mPI model, the model itself is complex and therefore has yet been only applied to systems with one DOF under controlled conditions. In this paper, a commercial micro-positioning unit is utilized to demonstrate the capabilites of this approach for two DOF. In Sec. 2, the mPI model and its corresponding cost function for optimization of the weights are briefly outlined. Experimental results are presented and analyzed in Sec. 3 and Sec. 4 concludes the paper.

2 Modified Prandtl-Ishlinskii Model, its Inverse, and Derived Optimization Problem

The mPI model is comprised of the play operator $\boldsymbol{H}_{\boldsymbol{r}_H}$, the superposition operator $\boldsymbol{S}_{\boldsymbol{r}_S}$, and the creep operator $\boldsymbol{K}_{\boldsymbol{r}_K}$ which are each parameterized by a threshold $(\boldsymbol{r}_H, \boldsymbol{r}_S, \boldsymbol{r}_C)$ and model hysteresis, asymmetric hysteresis, and creep, respectively. A user-defined number of operators is weighted yielding the overall mPI operator $\Gamma = \mathbf{w}_S^T \mathbf{S}_{\mathbf{r}_S}(\mathbf{w}_H^T \mathbf{H}_{\mathbf{r}_H} + \mathbf{w}_K^T \mathbf{K}_{\mathbf{r}_K})$ with corresponding weights $\mathbf{w}_H$, $\mathbf{w}_S$, and $\mathbf{w}_K$. As outlined in Sec. 1, the mPI approach offers an inverse which is directly obtained by parameter transformation of the thresholds and weights (denoted by $(.)'$) such that the inverse mPI model is $\Gamma^{-1} = \mathbf{w}_H^{\prime T} \mathbf{H}_{\mathbf{r}'_H}(\mathbf{w}_S^{\prime T} \mathbf{S}_{\mathbf{r}'_S} - \mathbf{w}_K^T \mathbf{K}_{\mathbf{r}_K})$. The squared error between model output and actuator output can be rearranged into linear form $E^2(k) = \mathbf{w}^T \mathbf{\Phi}(k) \mathbf{\Phi}^T(k) \mathbf{w} + 2x(k) \mathbf{\Phi}^T(k) \mathbf{w} + x^2(k)$, where $\mathbf{w}$ comprises the weights, $\mathbf{\Phi}$ the operators, and $x(k)$ is the compensator output at each discretized point k. From N_t recorded databases (each consisting of N_k discrete data points), a cost function V based on the squared error in its discretized form [3] can be formed by

$$V = \frac{1}{2} \sum_{i=0}^{N_t-1} \sum_{k=iN_k}^{(i+1)N_k} E^2(k) = \sum_{i=0}^{N_t-1} \left\{ \frac{1}{2} \mathbf{w}^T \underbrace{\sum_{k=iN_k}^{(i+1)N_k} \mathbf{\Phi}(k) \mathbf{\Phi}^T(k)}_{=:\mathbf{A}_i} \mathbf{w} + \underbrace{\sum_{k=iN_k}^{(i+1)N_k} x(k) \mathbf{\Phi}^T(k)}_{=:\mathbf{b}_i^T} \mathbf{w} + \frac{1}{2} \sum_{k=iN_k}^{(i+1)N_k} x^2(k) \right\}. \quad (1)$$

A quadratic optimization problem with corresponding constraints can be derived from (1) yielding the corresponding weights for online adaptation.

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[*] Corresponding Author: email christopher.schindlbeck@imr.uni-hannover.de, phone +49 511 762 3235, fax +49 511 762 3234

[**] christian.pape@imr.uni-hannover.de

[***] eduard.reithmeier@imr.uni-hannover.de

3 Experimental Results

Fig. 1: Piezoelectric micro-positioning unit driven by piezoactuators (left). Each piezoelectric actuator is preloaded by an elliptic spring shell (middle). An X-Y cross section exposes the antagonistic design for hysteretic asymmetry cancellation (right).

The 3-DOF micro-positioning unit *Cedrat Technologies XYZ200M* driven by piezoelectric actuators is chosen for the experimental validation, see Fig. 1 (left). Each actuator (see Fig. 1 (middle)) is preloaded by an external elliptical spring shell made of stainless steel which not only amplifies the displacement of the piezoelectric crystals by a factor of five but also protects e.g. against tensile stress. The X- and Y-axes follow antagonistic design, i.e. two opposing piezoelectric actuators for each axis, respectively (see Fig. 1 (right)) which enhances point-symmetry of the hysteresis curve but also induces disturbances due to mechanical coupling. The micro-positioning unit has a nominal displacement of 200 μm and nanoscopic resolution of 2 nm. We command the voltage input and access strain-gauge measurements via a *National Instruments* real-time system which can be mapped to displacement after appropriate calibration.
To demonstrate 2-DOF capabilities of the online recursive algorithm, two different trajectories are generated, see Fig. 2. Tracking performance based on the normalized root mean square error is improved by at least 70 % for each axis.

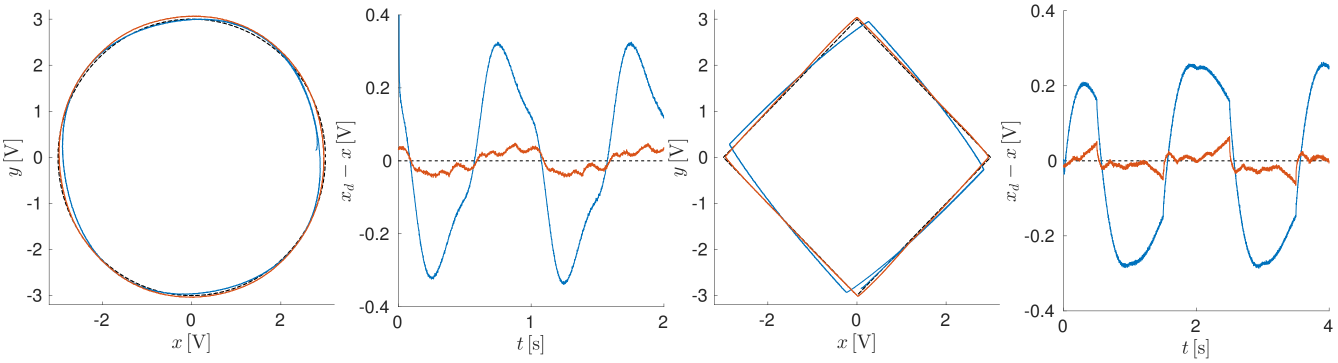

Fig. 2: Two coupled trajectories in the X-Y plane with reference (black, dashed), without compensation (blue), and with compensation (red). Circular trajectory (left) and rectangular trajectory (right) with corresponding tracking error for the X-axis.

4 Conclusion & Future Work

2-DOF feedforward control with weight adaptation for piezoelectric actuators has shown to improve tracking in non-contact scenarios. However, external forces e.g. due to contact with constrained environments during manipulation tasks necessitates disturbance rejection techniques which can be addressed in future works by augmenting the online feedforward control with an additional feedback control loop (based on sensors or an observer).

References

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