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Structural vibration control

E. Reithmeier[a,]*, G. Leitmann[b]

[a] *Institut für Meß- und Regelungstechnik, Universität Hannover, Nienburger Str. 17, 30167 Hannover, Germany*

[b] *College of Engineering, University of California at Berkeley, USA*

Abstract

Undesirable time-variable motions of dynamical structures (e.g. scales, balances, vibratory platforms, bridges and buildings) are mainly caused by unknown or uncertain excitations. In a variety of applications it is desirable or even necessary to attenuate these disturbances in an effective way and with moderate effort. Hence, several passive as well as active methods and techniques have been developed in order to treat these problems. However, employment of active techniques often fails because of their considerable financial costs. We propose an affordable control scheme which accounts for the above-mentioned deficiencies. In addition, we allow constraints on control actions. Furthermore, the number of control inputs (actuators) may be arbitrary, i.e., the system may be mismatched. The scheme is based on Lyapunov stability theory and, provided that the bounds of the uncertainties are a priori known, a stable attractor (ball of ultimate boundedness) of the structure can be computed. In case measurement errors or uncertainties, respectively, are significant, it is shown how the Lyapunov-based control scheme may be combined with a fuzzy control concept. The effectiveness and behavior of the control scheme is demonstrated on two simplified models of elastic structures such as a two story building and a bridge subjected to a moving truck.

Keywords: Vibrations; Structures; Robust control; Lyapunov stability; Fuzzy theory; Identification

1. Introduction

The class of systems which we shall take into consideration may be described by a dynamical system with a finite number of degrees of freedom. The structure has to

*Corresponding author. Fax: +49-511-762-3234.

E-mail address: sekretariat@imr.uni-hannover.de (E. Reithmeier).

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contain "active" elastic coupling elements. We call suspension or coupling elements "active" if they are adjustable with respect to their stiffness and damping behavior. Based on this model, a control action is related to a change in these properties. The mathematical description of these kinds of structures is assumed to be an initial value problem of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(\mathbf{x}, \mathbf{z}) \cdot \mathbf{u} + \mathbf{e}(\mathbf{x}, \mathbf{z}, t), \quad \mathbf{x}(0) = \mathbf{x}_0. \tag{1}$$

The linear part of the mathematical model of the structure is defined by the constant and stable matrix $\mathbf{A} \in \mathbb{R}^{n,n}$, where $n \in \mathbb{N}$ denotes the state space dimension. The control input matrix $\mathbf{B}(\mathbf{x}, \mathbf{z}) \in \mathbb{R}^{n,m}$ is assumed to be continuous in $\mathbf{x}$ and $\mathbf{z}$. $\mathbf{x} \in \mathbb{R}^n$ represents the n state variables; $\mathbf{z} \in \mathbb{R}^r$ the r "disturbances" whose current values can be measured, $\mathbf{u} \in \mathbb{R}^m$ the m control variables. Furthermore, we will assume that

$$rk[\mathbf{B}(\mathbf{x}, \mathbf{z})] = m \quad \forall (\mathbf{x}, \mathbf{z}) \in \mathbb{R}^n \times \mathbb{R}^r. \tag{2}$$

holds. That is, we assume that the control variables are not redundant or, in other words, every control variable acts in a different direction. The control variables are supposed to be constrained by lower and upper bounds

$$\mathscr{U} := [u_{1,\min}, u_{1,\max}] \times \cdots \times [u_{m,\min}, u_{m,\max}]. \tag{3}$$

This restriction reflects the fact that applied control actions in practice are restricted because of technical reasons. Without loss of generality and for the sake of convenience, we may assume that

$$\mathscr{U} = [-1, 1] \times \cdots \times [-1, 1]. \tag{4}$$

All unknown or uncertain excitations (including unmodelled non-linearity) are modelled by

$$\mathbf{e}\colon \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R} \to \mathscr{B}_\varepsilon := \{\xi \in \mathbb{R}^n \mid \|\xi\| \leqslant \varepsilon \in \mathbb{R}_+\}. \tag{5}$$

That is, all uncertainties are assumed to be bounded by the compact ball $\mathscr{B}_\varepsilon$ with radius ε:

$$\|\mathbf{e}(\mathbf{x}, \mathbf{z}, t)\| \leqslant \varepsilon \in \mathbb{R}_+ \quad \forall (\mathbf{x}, \mathbf{z}, t) \in \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R}_+. \tag{6}$$

We assume also that $\mathbf{e}$ is piecewise continuous. Furthermore, we assume that the state $\mathbf{x}$ as well as the disturbances $\mathbf{z}$ are available via some measurement devices. In case of seismic disturbances, $\mathbf{z}$ may be determined by seismograph records. That is, $\mathbf{x}$ and $\mathbf{z}$ are considered as plant output $\mathbf{y} := (\mathbf{x}, \mathbf{z})$ (see Fig. 1).

2. Control design

2.1. Lyapunov approach

Following the controller design in Leitmann and Reithmeier [1] we ask for a feedback

$$\mathbf{p}\colon \mathbb{R}^n \times \mathbb{R}^r \to \mathscr{U}, \quad (\mathbf{x}, \mathbf{z}) \mapsto \mathbf{u} = (\mathbf{p}(\mathbf{x}, \mathbf{z})) \tag{7}$$

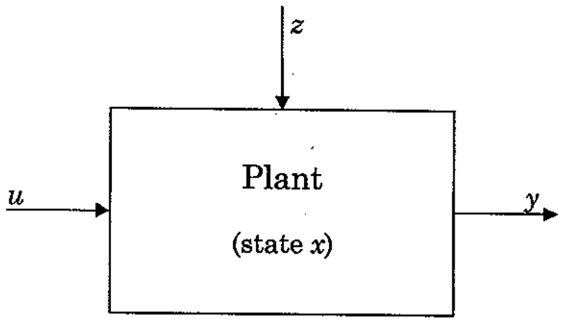

Fig. 1. Input and output variables of the plant.

such that, for any given positive definite matrix $\mathbf{P} \in \mathbb{R}^{n,n}$, the time derivative of

$$\|\mathbf{x}(t)\|_P := \sqrt{\mathbf{x}^{\mathrm{T}}(t)\mathbf{P}\mathbf{x}(t)}, \quad t \in \mathbb{R}_+ \tag{8}$$

is as small as possible for any:

(i) response $\mathbf{x}(t)$,
(ii) disturbance $\mathbf{z}(t)$,
(iii) admissible uncertainty $\mathbf{e}(\mathbf{x}(t), \mathbf{z}(t), t)$,
(iv) and time t,

and for all admissible choices of control $\mathbf{u}(t)$.

This means that, based on the Lyapunov function candidate

$$V(\mathbf{x}) := \tfrac{1}{2}\|\mathbf{x}\|_P^2, \tag{9}$$

the feedback $\mathbf{p}(\mathbf{x}, \mathbf{z})$ we are seeking is one which minimizes the Lyapunov derivative

$$\begin{aligned}\mathscr{L}_{(\mathbf{x},\mathbf{z},t)}(\mathbf{u}) &:= \mathbf{x}^{\mathrm{T}}\mathbf{P}[\mathbf{A}\mathbf{x} + \mathbf{B}(\mathbf{x},\mathbf{z})\mathbf{u} + \mathbf{e}(\mathbf{x},\mathbf{z},t)] \\ &= \mathbf{x}^{\mathrm{T}}\mathbf{P}\left[\mathbf{A}\mathbf{x} + \sum_{j=1}^{m} u_j \mathbf{B}(\mathbf{x},\mathbf{z})\mathbf{i}_j + \mathbf{e}(\mathbf{x},\mathbf{z},t)\right]\end{aligned} \tag{10}$$

with respect to $\mathbf{u}$ for every $(\mathbf{x}, \mathbf{z}, t) \in \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R}_+$, where $\mathbf{i}_j$ is the unit vector in the j direction, $\mathbf{i}_i^{\mathrm{T}}\mathbf{i}_j = 0$ for $i \neq j$. In other words, we seek $\mathbf{p}(\mathbf{x}, \mathbf{z})$ such that

$$\mathscr{L}_{(\mathbf{x},\mathbf{z},t)}(\mathbf{p}(\mathbf{x},\mathbf{z})) = \min\{\mathscr{L}_{(\mathbf{x},\mathbf{z},t)}(\mathbf{u}) \mid \mathbf{u} \in \mathscr{U}\}. \tag{11}$$

Eq. (10) can be written as

$$\mathscr{L}_{(\mathbf{x},\mathbf{z},t)}(\mathbf{u}) = a(\mathbf{x}) + \sum_{j=1}^{m} u_j b_j(\mathbf{x},\mathbf{z}) + c(\mathbf{x},\mathbf{z},t) \tag{12}$$

with

$$\begin{aligned}a(\mathbf{x}) &:= -\tfrac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} \quad \text{where } \mathbf{Q} := -(\mathbf{P}\mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{P}), \\ b_j(\mathbf{x},\mathbf{z}) &:= \mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{B}(\mathbf{x},\mathbf{z})\mathbf{i}_j \quad \text{where } \mathbf{i}_j\mathbf{i}_k = \delta_{jk}\end{aligned} \tag{13}$$

and

$$c(\mathbf{x}, \mathbf{z}, t) := \mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{e}(\mathbf{x}, \mathbf{z}, t). \tag{14}$$

Then

$$u_j = p_j(\mathbf{x}, \mathbf{z}) = \begin{cases} u_{j,\min} & \text{if } b_j(\mathbf{x}, \mathbf{z}) > 0, \\ u_{j,\max} & \text{if } b_j(\mathbf{x}, \mathbf{z}) < 0. \end{cases} \quad (j = 1, \ldots, m) \tag{15}$$

Using the normalized control space $\mathscr{U}$ defined by Eq. (4) we obtain

$$u_j = p_j(\mathbf{x}, \mathbf{z}) = -\mathrm{sgn}[b_j(\mathbf{x}, \mathbf{z})]. \tag{16}$$

The performance of the controller may be enhanced additionally if we choose $\mathbf{P}$ appropriately. The smaller the Lyapunov derivative, the stronger the "tendency to the origin" of $t \mapsto \mathbf{x}(t)$. That, on the other hand, can be done by choosing a suitable positive definite $\mathbf{Q}$ and solving the algebraic Lyapunov equation

$$\mathbf{Q} = -(\mathbf{P}\mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{P}) \tag{17}$$

for $\mathbf{P}$. Since $\mathbf{A}$ is assumed to be stable, the matrix $\mathbf{P}$ is positive definite.

2.2. Stability

Using the control scheme developed in Chapter 2.1 it is possible to determine a region (compact set) toward which any realization $t \mapsto \mathbf{x}(t)$ is attracted and wherein it remains once it has crossed the boundary of the "attractor". In order to analyze this situation we employ the controller $\mathbf{p}$ (cf. Eq. (16)) in expression (12). This leads to

$$\mathscr{L}_{(\mathbf{x},\mathbf{z},t)}(\mathbf{p}(\mathbf{x}, \mathbf{z})) = -\tfrac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} - \sum_{j=1}^{m} |b_j(\mathbf{x}, \mathbf{z})| + \mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{e}(\mathbf{x}, \mathbf{z}, t), \tag{18}$$

where $\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x}$ is a positive definite quadratic form. Hence, the form is bounded by the minimum and maximum eigenvalue $\lambda_{\min}(\mathbf{Q})$ and $\lambda_{\max}(\mathbf{Q})$ of $\mathbf{Q}$. That is,

$$\mathscr{L}_{(\mathbf{x},\mathbf{z},t)}(\mathbf{p}(\mathbf{x}, \mathbf{z})) \leqslant -\tfrac{1}{2}\lambda_{\min}(\mathbf{Q})\|\mathbf{x}\|^2 - \sum_{j=1}^{m} |b_j(\mathbf{x}, \mathbf{z})| + \|\mathbf{x}\|\,\|\mathbf{P}\|\,\|\mathbf{e}(\mathbf{x}, \mathbf{z}, t)\| \tag{19}$$

or, neglecting the second term, employing inequality (6) and using the maximum eigenvalue $\lambda_{\max}(\mathbf{P})$ of $\mathbf{P}$

$$\mathscr{L}_{(\mathbf{x},\mathbf{z},t)}(\mathbf{p}(\mathbf{x}, \mathbf{z})) \leqslant -\tfrac{1}{2}\lambda_{\min}(\mathbf{Q})\|\mathbf{x}\|^2 + \lambda_{\max}(\mathbf{P})\varepsilon\|\mathbf{x}\|. \tag{20}$$

Thus, we obtain

$$\mathscr{L}_{(\mathbf{x},\mathbf{z},t)}(\mathbf{p}(\mathbf{x}, \mathbf{z})) \leqslant 0 \quad \forall \|\mathbf{x}\| > r, \tag{21}$$

where

$$r := \frac{2\varepsilon\lambda_{\max}(\mathbf{P})}{\lambda_{\min}(\mathbf{Q})}. \tag{22}$$

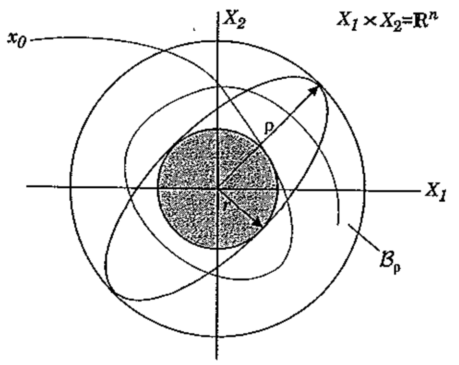

Fig. 2. Ball of ultimate boundedness.

The radius ρ of a *ball of ultimate boundedness* (cf. Fig. 2) is therefore determined by

$$\rho = r\sqrt{\frac{\lambda_{\max}(\mathbf{P})}{\lambda_{\min}(\mathbf{P})}} \tag{23}$$

and any response $t \mapsto \mathbf{x}(t)$ which enters

$$\mathcal{B}_\rho := \{\xi \in \mathbb{R}^n \mid \|\xi\| \leqslant \rho\} \tag{24}$$

say at $t = t^*$ remains in $\mathcal{B}_\rho$ for all $t > t^*$. It should be noted that $\mathcal{B}_\rho$, with radius given in (23), is a ball of ultimate boundedness for control actions $u_j = 0\ \forall j$, since the terms $\sum_{j=1}^{m} |b_j(\mathbf{x}, \mathbf{z})|$ were neglected in (19). These terms reduce the right-hand side of (19), except at $(\mathbf{x}, \mathbf{z})$ where the $b_j(\mathbf{x}, \mathbf{z}) = 0$ for *all* j. In other words, the control acts to reduce the radius of an *actual* ball of ultimate boundedness (that is, one for the controlled system) as well as the rate of convergence.

3. Measurement errors

3.1. General aspects

A change in the control variables u_j takes place only if the *indicator function* b_j, defined in (13) changes its sign. That is, a change in u_j occurs if a response $t \mapsto \mathbf{y}(t) = (\mathbf{x}(t), \mathbf{z}(t))$ crosses

$$\Pi_j := \{(\mathbf{x}, \mathbf{z}) \in \mathbb{R}^n \times \mathbb{R}^r \mid b_j(\mathbf{x}, \mathbf{z}) = 0\}. \tag{25}$$

We assume that Π_j denotes a differentiable manifold of dimension $n + r - 1$; otherwise the space $\mathbb{R}^n \times \mathbb{R}^m$ would not be separated by Π_j into two disjoint parts. Furthermore, then

$$\mathbf{g}_j\colon\ \Pi_j \to \mathbb{R}^n \times \mathbb{R}^r, \quad \xi \mapsto \mathbf{y} := \mathbf{g}_j(\xi) \tag{26}$$

defines a unique and differentiable embedding of Π_j into $\mathbb{R}^n \times \mathbb{R}^r$ (cf. Fig. 3).

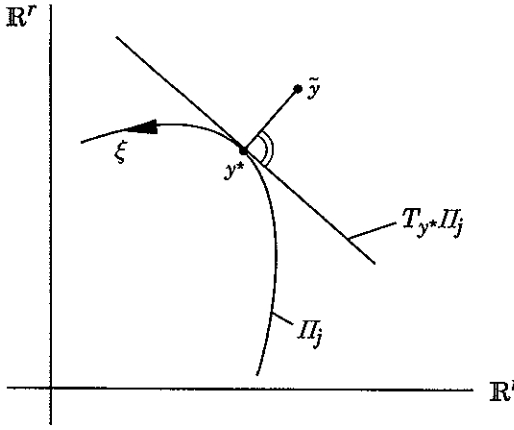

Fig. 3. Embedding of Π_j in $\mathbb{R}^n \times \mathbb{R}^r$.

3.2. Fuzzy concept

Depending on the accuracy of the measurement device there is always uncertainty in the measured variables. The assumed maximum difference between actual value $\mathbf{y}$ and measured value $\tilde{\mathbf{y}}$ may be expressed by $\Delta\mathbf{y} := (\Delta x_1, \ldots, \Delta x_n, \Delta z_1, \ldots, \Delta z_r)^{\mathrm{T}}$, such that

$$\|\mathbf{y} - \tilde{\mathbf{y}}\| \leqslant \|\Delta\mathbf{y}\|_M := 1. \tag{27}$$

$\|\cdot\|_M$: $\mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}_+$ is some norm which takes care of the tolerances and scales in measurement for the different variables. For instance, it might be defined by

$$\|\mathbf{y} - \tilde{\mathbf{y}}\|_M := \sqrt{\left(\frac{1}{n+r}\right)\left[\sum_{i=1}^{n}\left(\frac{x_i - \tilde{x}_i}{\Delta x_i}\right)^2 + \sum_{i=1}^{r}\left(\frac{z_i - \tilde{z}_i}{\Delta z_i}\right)^2\right]}. \tag{28}$$

As mentioned before, a change in control u_j will occur only on the switching plane Π_j for $j = 1, \ldots, m$. Hence, we define uncertain transition regions $\mathcal{T}_j$ (cf. Fig. 4) as follows: if the measured value $\tilde{\mathbf{y}}$ indicates switching, then the actual value $\mathbf{y}$ belongs to the transition area, that is

$$\tilde{\mathbf{y}} \in \Pi_j \;\Rightarrow\; \mathbf{y} \in \mathcal{T}_j. \tag{29}$$

The distance between $\tilde{\mathbf{y}}$ and Π_j is taken to be the perpendicular to the tangent plane $T_{\mathbf{y}^*}\Pi_j$ at an appropriate point $\mathbf{y}^* \in \Pi_j$ (cf. Fig. 3). According to that definition, $\mathbf{y}^*$ is determined by solving the following algebraic equation:

$$[D\mathbf{g}_j(\xi^*)]^{\mathrm{T}}(\tilde{\mathbf{y}} - \mathbf{g}_j(\xi^*)) = 0 \;\Rightarrow\; \xi^*(\tilde{\mathbf{y}}) \;\Rightarrow\; \mathbf{y}^* = \mathbf{g}_j(\xi^*(\tilde{\mathbf{y}})). \tag{30}$$

Using Eq. (29) we are able to define $\mathcal{T}_j$ via

$$\mathcal{T}_j := \{\hat{\mathbf{y}} \in \mathbb{R}^n \times \mathbb{R}^r \mid \|\hat{\mathbf{y}} - \mathbf{g}_j(\xi^*(\hat{\mathbf{y}}))\|_M \leqslant 1\}. \tag{31}$$

In order to replace the designed Lyapunov controller on $\mathcal{T}_j$ by some appropriate fuzzy controller we use relationship (30) as a *fuzzyfication* process. Within $\mathcal{T}_j$ the

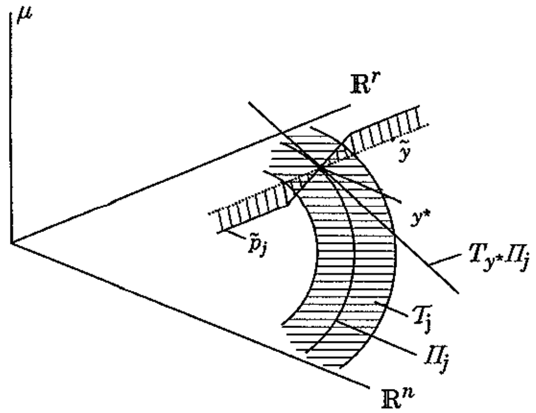

Fig. 4. Uncertain transition area $\mathscr{T}_j$ and membership function μ_j.

choice of the control variables may be right or may be wrong. If the decision is right the choice will be the best case, if the decision is wrong the choice will be the worst case. Therefore it seems very reasonable to *defuzzify* the transition area in such a way that there exists a linear relationship (membership function) within $\mathscr{T}_j$. Or, in terms of probability, we assume a linear distribution across $\mathscr{T}_j$, which says that the chance for $\tilde{\mathbf{y}}$ to be right or to be wrong is 50% on the switching surfaces and 100% on the border of $\mathscr{T}_j$. In between it is supposed to be linearly increasing or decreasing, respectively (cf. Fig. 4). Hence, the membership function is defined by

$$\mu\colon \mathscr{T}_j \to \mathscr{U}, \quad \tilde{\mathbf{y}} \mapsto \mu(\tilde{\mathbf{y}}) \tag{32}$$

with

$$\mu_j(\tilde{\mathbf{y}}) := \begin{cases} 0 & \text{if } \tilde{\mathbf{y}} \in \Pi_j, \\ \|\tilde{\mathbf{y}} - \mathbf{y}^*\|_M \cdot \operatorname{sgn}[b_j(\tilde{\mathbf{y}})] & \text{if } \tilde{\mathbf{y}} \in \Pi_j - \mathscr{T}_j. \end{cases} \quad (j = 1, \ldots, m) \tag{33}$$

Of course, for each j there exists a different $\mathbf{y}^*$ which needs to be determined via Eq. (30). The modified continuous state feedback is then given by

$$\tilde{p}_j(\tilde{\mathbf{y}}) := \begin{cases} \mu_j(\tilde{\mathbf{y}}) & \text{if } \|\tilde{\mathbf{y}} - \mathbf{y}^*\|_M < 1, \\ \operatorname{sgn}[b_j(\tilde{\mathbf{y}})] & \text{if } \|\tilde{\mathbf{y}} - \mathbf{y}^*\|_M \geqslant 1. \end{cases} \tag{34}$$

4. Examples

4.1. Structure with two stories

An example of the class of systems modelled by (1) is a structure with two stories as shown in Fig. 5. We will consider this example in order to demonstrate the efficacy and robustness of the proposed control scheme. In that case, the spring and damping

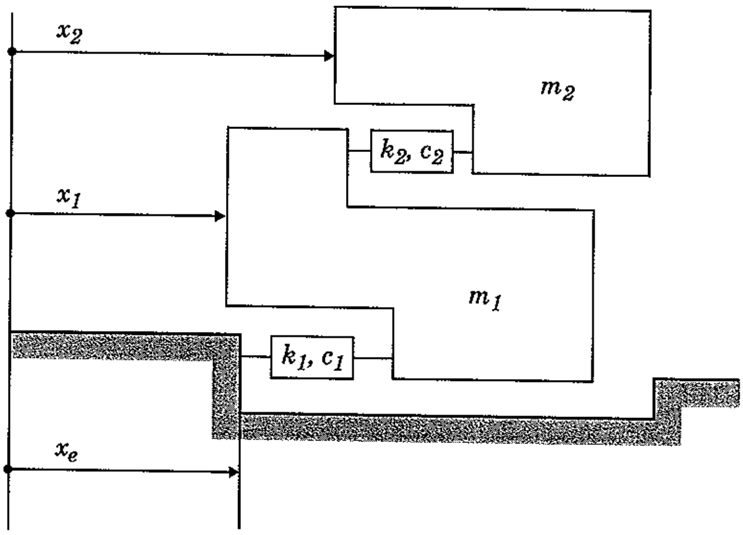

Fig. 5. Structure with two stories.

coefficients k_i and c_i are linear functions of the applied control action v_i which, for instance, could be the voltage applied on suspension elements filled with the so-called smart materials:

$$k_j(v_j) = \alpha_j^k + \beta_j^k v_j \quad (j = 1, 2),$$

$$c_j(v_j) = \alpha_j^c + \beta_j^c v_j \quad (j = 1, 2), \tag{35}$$

where α_j^k, α_j^c, β_j^k and $\beta_j^c \in \mathbb{R}_+$ are constant parameters and the voltage v_j for spring/damper j can be varied between 0 and $\bar{v}_j > 0$. The linear parameter shift

$$u_j := \left[2\left(\frac{v}{\bar{v}_j}\right) - 1\right] \tag{36}$$

transforms v_j into the normalized control variable u_j. For subsequent simulations, the realization of the disturbance $\mathbf{z}$ considered here is a periodic ground displacement

$$t \mapsto x_e(t) := \hat{x}_e \sin[\nu t]. \tag{37}$$

The equation of motion is given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(\mathbf{x}, \mathbf{z})\mathbf{u} + \mathbf{e}(\mathbf{x}, \mathbf{z}, t) \tag{38}$$

with

$$\mathbf{A} := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{1}{m_1}[\alpha_1^k + \alpha_2^k] & \frac{\alpha_2^k}{m_1} & -\frac{1}{m_1}[\alpha_1^c + \alpha_2^c] & \frac{\alpha_2^c}{m_1} \\ \frac{\alpha_2^k}{m_2} & -\frac{\alpha_2^k}{m_2} & \frac{\alpha_2^c}{m_2} & -\frac{\alpha_2^c}{m_2} \end{bmatrix} \tag{39}$$

and

$$\mathbf{B}(\mathbf{x},\mathbf{z}) := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\frac{\beta_1^k}{m_1}(x_1 - x_e) - \frac{\beta_1^c}{m_1}(x_3 - \dot{x}_e) & \frac{\beta_2^k}{m_1}(x_2 - x_1) + \frac{\beta_2^c}{m_1}(x_4 - x_3) \\ 0 & \frac{\beta_2^k}{m_2}(x_1 - x_2) + \frac{\beta_2^c}{m_2}(x_3 - x_4) \end{bmatrix}, \tag{40}$$

$$\mathbf{e}(\mathbf{x},\mathbf{z},t) := \begin{bmatrix} 0 \\ 0 \\ \frac{\alpha_1^k}{m_1}x_e + \frac{\alpha_1^c}{m_1}\dot{x}_e \\ 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} := \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} := \begin{bmatrix} x_e \\ \dot{x}_e \end{bmatrix}.^{1} \tag{41}$$

The indicator function b_j may be expressed by

$$b_j(\mathbf{y}) = \mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{B}_j\mathbf{y} \quad (j = 1,2), \tag{42}$$

where

$$\mathbf{y} := \begin{bmatrix} \mathbf{x} \\ \mathbf{z} \end{bmatrix} \quad \text{and} \quad \mathbf{B}_j\mathbf{y} := \mathbf{B}(\mathbf{x},\mathbf{z}) \cdot \mathbf{i}_j,$$

where $\mathbf{B}_j \in \mathbb{R}^{n,n+r}$ are constant matrices, with

$$rk[\mathbf{B}_j] = 1 \tag{43}$$

for $j = 1$ and 2. Hence, an appropriate Householder transformation $\mathbf{H}_j \in \mathbb{R}^{n,n}$ leads to

$$\begin{aligned} b_j(\mathbf{y}) &:= (\mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{H}_j^{\mathrm{T}}) \cdot (\mathbf{H}_j\mathbf{B}_j\mathbf{y}) \\ &= (\mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{H}_j^{\mathrm{T}}\mathbf{e}_1) \cdot (\mathbf{c}^{\mathrm{T}}\mathbf{y}), \end{aligned} \tag{44}$$

where

(i) $\mathbf{H}_j^{-1} = \mathbf{H}_j^{\mathrm{T}}$,

(ii) $\mathbf{H}_j\mathbf{B}_j\mathbf{y} = (\mathbf{c}^{\mathrm{T}}\mathbf{y})\mathbf{e}_1$,

(iii) $\mathbf{c} = const. \in \mathbb{R}^{n+r}, \quad \mathbf{e}_1 := (1,0,\ldots,0)^{\mathrm{T}} \in \mathbb{R}^n$.

Taking this into consideration, we obtain finally the following statement:

$$b_j(\mathbf{y}) = 0 \Leftrightarrow \mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{H}_j^{\mathrm{T}}\mathbf{e}_1 = 0 \quad \text{and/or } \mathbf{c}^{\mathrm{T}}\mathbf{y} = 0 \tag{45}$$

i.e., the manifold Π_j degenerates into two planes whose intersection contains the origin $\mathbf{0}$ (cf. Fig. 6). This means, in particular, that

$$T_{\mathbf{y}}\Pi_j = \Pi_j \quad \forall \mathbf{y} \in \Pi_j. \tag{46}$$

[1] In earlier work, e.g. (Kelly et al. [2]) or (Leitmann [3]), it is assumed that $x_e, \dot{x}_e$ are not measured but that their bounds are known; in that event, x_e and $\dot{x}_e$ contribute to uncertainty in $\mathbf{e}(\mathbf{x},\mathbf{z},t)$.

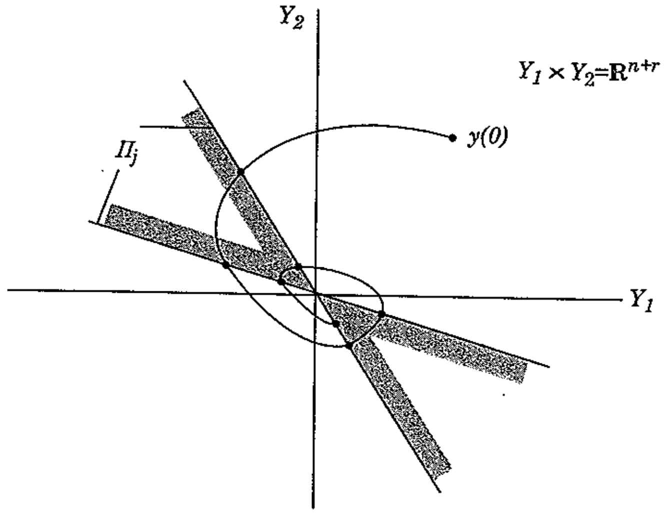

Fig. 6. Switching plane Π_j for u_j.

The following simulations are for initial value problems with $\mathbf{x}_0 = \mathbf{0}$ (cf. Eq. (1)). The dynamical system is integrated for 15 s and an "amplitude" of each state component x_i is defined as the maximum of its absolute value during the final 5 s.

All results are based on the parameter set ($i = 1, 2$):

$$\bar{v}_i = 10^3 \text{ [V]}, \quad m_i = 10 \text{ [kg]}, \quad \hat{x}_e = 0.02 \text{ [m]} \tag{47}$$

and

$$\alpha_i^c = 4\left[\frac{\text{Ns}}{\text{m}}\right], \quad \beta_i^c = 8\left[\frac{\text{Ns}}{\text{Vm}}\right], \quad \alpha_i^k = 2\times 10^3\left[\frac{\text{N}}{m}\right], \quad \beta_i^k = 10^3\left[\frac{\text{N}}{\text{Vm}}\right]. \tag{48}$$

The fuzzy controller is defined according to Eq. (34) with

$$\|\mathbf{y}-\tilde{\mathbf{y}}\|_M :=$$

$$\sqrt{\left(\frac{1}{4+2}\right)\left[\left(\frac{x_1-\tilde{x}_1}{\Delta x_1}\right)^2+\left(\frac{x_2-\tilde{x}_2}{\Delta x_2}\right)^2+\left(\frac{\dot{x}_1-\dot{\tilde{x}}_1}{\Delta \dot{x}_1}\right)^2+\left(\frac{\dot{x}_2-\dot{\tilde{x}}_2}{\Delta \dot{x}_2}\right)^2+\left(\frac{x_e-\tilde{x}_e}{\Delta x_e}\right)^2+\left(\frac{\dot{x}_e-\dot{\tilde{x}}_e}{\Delta \dot{x}_e}\right)^2\right]}, \tag{49}$$

where Δx_1, Δx_2, $\Delta \dot{x}_1$, $\Delta \dot{x}_2$, Δx_e and $\Delta \dot{x}_e$ are the maximum errors which may occur during measurement. They are given by

$$\Delta\mathbf{x} := \begin{bmatrix}\Delta x_1\\ \Delta x_2\\ \Delta \dot{x}_1\\ \Delta \dot{x}_2\end{bmatrix} = \begin{bmatrix}5\times 10^{-3}[\text{m}]\\ 5\times 10^{-3}[\text{m}]\\ 0.1[\text{m/s}]\\ 0.1[\text{m/s}]\end{bmatrix} \quad \text{and} \quad \Delta\mathbf{z} := \begin{bmatrix}\Delta x_e\\ \Delta \dot{x}_e\end{bmatrix} = \begin{bmatrix}5\times 10^{-3}[\text{m}]\\ 0.1[\text{m/s}]\end{bmatrix}. \tag{50}$$

The results are shown in Figs. 7–10. Each diagram shows the four different cases

- only the Lyapunov controller is used,
- the Lyapunov and Fuzzy controller are combined,
- constant maximum damping and maximum stiffness is applied ($u_j = u_{j,\max}$),
- no control at all is applied ($u_j = 0$).

As one can easily recognize, there is a significant suppression for all state components x_i in the controlled cases. The cases "constant minimum stiffness and damping" and "constant maximum stiffness and damping" are significantly worse

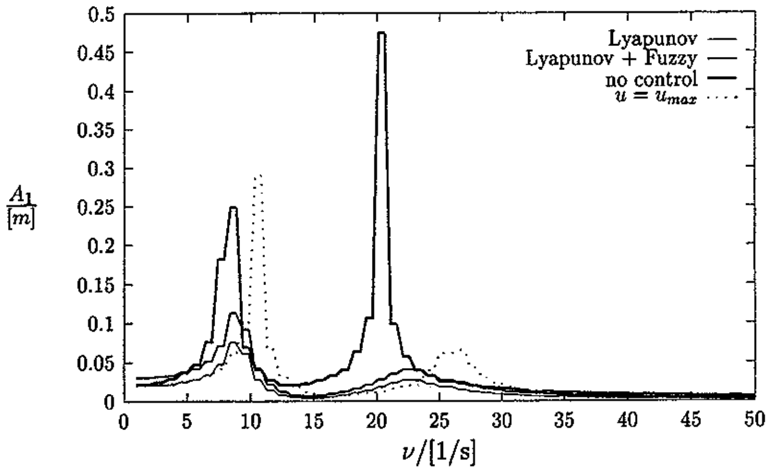

Fig. 7. Maximum amplitude A_1 of x_1 versus excitation frequency ν.

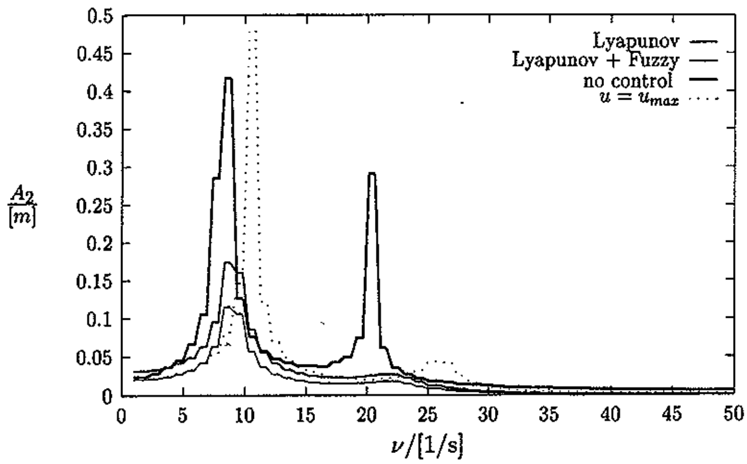

Fig. 8. Maximum amplitude A_2 of x_2 versus excitation frequency ν.

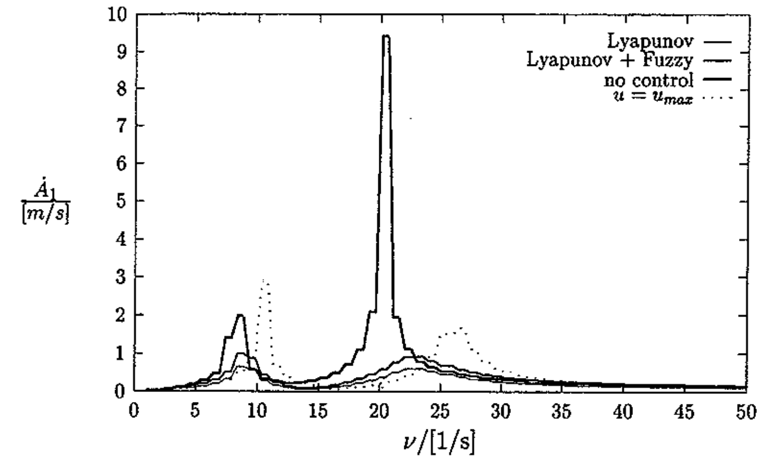

Fig. 9. Maximum amplitude $\dot{A}_1$ of $\dot{x}_1$ versus excitation frequency ν.

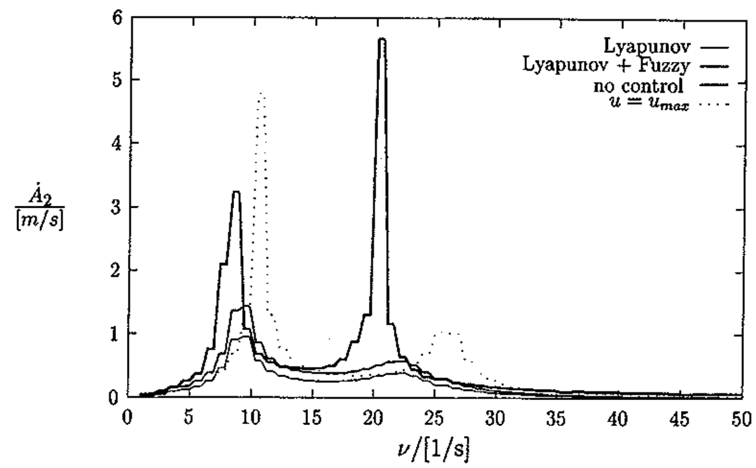

Fig. 10. Maximum amplitude $\dot{A}_2$ of $\dot{x}_2$ versus excitation frequency ν.

near their resonance frequencies. Although, the "pure Lyapunov controller" leads to the best results, it does not account for measurement errors. Taking into consideration that assumed measurement errors may rise up to 25% of the ground displacement x_e and up to 50% for low frequencies ν, the combined "Lyapunov + Fuzzy" control seems to be a very reasonable choice.

Of course, the smaller the Euclidian norm $||\Delta \mathrm{y}||$ of the maximum measurement errors Δy_i the smaller the transition areas $\mathscr{T}_j$ around the switching planes Π_j and the smaller the difference between the Lyapunov controller and the combined "Lyapunov + Fuzzy" control. It is important to note that the significant suppression of time-variable displacements of the state components x_i takes place in resonant frequency ranges, and these are exactly the frequency domains where suppression in most cases is really wanted. Furthermore, it is important to mention that any time delay during feedback control response may be modelled by an additional uncertainty in all measured state variables. It should be also mentioned at this point that a possible chattering effect along Π_j will be suppressed by adding the Fuzzy controller near the switching surface.

4.2. Bridge with crossing truck

Fig. 11 shows a simplified model of a bridge and a truck which crosses the bridge. The truck model consists of a mass m and an elastic chassis with wheel suspension (stiffness k and damping c). The truck moves with velocity v. m, k, c and v are considered to be uncertain since different trucks may cross the bridge. In this case, we do not measure any uncertainties or disturbances. The bridge is modelled by a

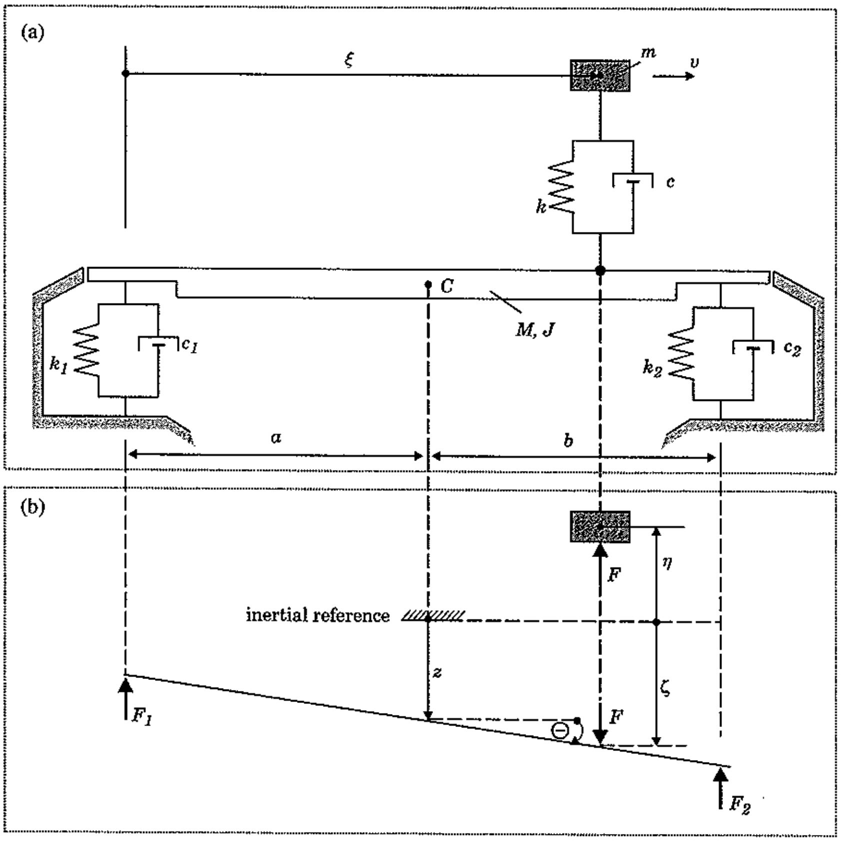

Fig. 11. Model of an actively mounted bridge with trucks crossing.

rigid platform with elastic mounts on the left- and right-hand sides. That is, we consider mainly the first two eigenmodes of the bridge, described by the deviation z of the center of mass C and the declination Θ with respect to the horizontal position of the bridge. The elastic mounts are considered to be active with stiffness

$$k_j(u_j) = \alpha_j^k + \beta_j^k u_j$$

and damping

$$c_j(u_j) = \alpha_j^c + \beta_j^c u_j$$

for $j = 1, 2$. u_j is the normalized and constrained control variable (see Example 4.1); i.e.,

$$u_j \in [-1,\ 1].$$

The mass of the bridge is given by the parameter M, the transverse moment of inertia by the parameter J. If we introduce the variables ξ, η and ζ according to Fig. 11, then we obtain the following equations of motion (Θ and z are assumed to be "small")

Truck:

$$m \cdot \ddot{\eta} = F - m \cdot g$$

with

$$F = k(\eta_0 - (\eta + \zeta)) - c(\dot{\eta} + \dot{\zeta}),$$

$$\eta_0 := \text{ position of relaxed truck suspension for } \xi = 0,$$

$$\zeta := z + (\xi - a) \cdot \Theta.$$

Bridge:

$$\text{(i)} \quad M\ddot{z} = Mg + F - F_1 - F_2,$$

$$\text{(ii)} \quad J\ddot{\Theta} = (\xi - a)F + aF_1 - bF_2$$

with

$$F_1 = k_1[-z_{1,0} + z - a\Theta] + c_1[\dot{z} - a\dot{\Theta}],$$

$$F_2 = k_2[-z_{2,0} + z + b\Theta] + c_2[\dot{z} + b\dot{\Theta}],$$

$$z_{1,0} := \text{vertical position of relaxed left-hand suspension},$$

$$z_{2,0} := \text{vertical position of relaxed right-hand suspension}.$$

The truck causes excitation of the bridge. However, compared to example 4.1, here we need to make sure, that there actually exists a ball of ultimate boundedness with respect to all state variables.

In order to approach that problem, we split the state variables in two parts, namely into the ones we measure

$$\mathbf{x} := (z, \Theta, \dot{z}, \dot{\Theta})^{\mathrm{T}}$$

and into the ones we do not measure

$$\mathbf{y} := (\eta, \dot{\eta})^{\mathrm{T}}$$

since they belong to the uncertain excitation. The equation of motion of the total system then has the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathbf{x}} = \hat{\mathbf{A}}\hat{\mathbf{x}} + \begin{bmatrix} \mathbf{B}(\mathbf{x})\mathbf{u} \\ \mathbf{0} \end{bmatrix} + \hat{\mathbf{e}}(\hat{\mathbf{x}}, t)$$

with

$$\hat{\mathbf{x}} := \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix},$$

$$\hat{\mathbf{A}} = (\hat{a}_{ij})_{\substack{i=1,\dots,6, \\ j=1,\dots,6}}$$

where

$$\hat{a}_{13} = 1,$$

$$\hat{a}_{24} = 1,$$

$$\hat{a}_{31} = -[\alpha_1^k + \alpha_2^k]/M,$$

$$\hat{a}_{32} = [a\alpha_1^k - b\alpha_2^k]/M,$$

$$\hat{a}_{33} = -[\alpha_1^c + \alpha_2^c]/M,$$

$$\hat{a}_{34} = [a\alpha_1^c - b\alpha_2^c]/M,$$

$$\hat{a}_{41} = [a\alpha_1^k - b\alpha_2^k]/J,$$

$$\hat{a}_{42} = -[a^2\alpha_1^k + b^2\alpha_2^k]/J,$$

$$\hat{a}_{43} = [a\alpha_1^c - b\alpha_2^c]/J,$$

$$\hat{a}_{44} = -[a^2\alpha_1^c + b^2\alpha_2^c]/J,$$

$$\hat{a}_{56} = 1,$$

$$\hat{a}_{65} = -k/m,$$

$$\hat{a}_{66} = -c/m,$$

and $\hat{a}_{ij} = 0$ for all other indices. The input matrix

$$\mathbf{B}(\mathbf{x}) = (b_{ij}(\mathbf{x}))_{\substack{i=1,\dots,4, \\ j=1,2}}$$

where

$$b_{31}(x) = [-\beta_1^k(z - z_{1,0} - a\cdot\Theta) - \beta_1^c(\dot{z} - a\dot{\Theta})]/M,$$

$$b_{32}(x) = [-\beta_2^k(z - z_{2,0} + b\cdot\Theta) - \beta_2^c(\dot{z} + b\dot{\Theta})]/M,$$

$$b_{41}(x) = [a\beta_1^k(z - z_{1,0} - a\cdot\Theta) + a\beta_1^c(\dot{z} - a\dot{\Theta})]/J,$$

$$b_{42}(x) = [-b\beta_2^k(z - z_{2,0} + b\cdot\Theta) - b\beta_2^c(\dot{z} + b\dot{\Theta})]/J$$

and $b_{ij}(\mathbf{x}) = 0$ for all other indices. Furthermore, we obtain

$$\hat{\mathbf{e}}(\hat{\mathbf{x}}, t) = \mathbf{C}(t)\hat{\mathbf{x}} + \gamma(t)$$

with

$$\mathbf{C}(t) = (c_{ij}(t))_{\substack{i=1,\ldots,6\\ j=1,\ldots,6}},$$

where

$$c_{31}(t) = -k/M,$$

$$c_{32}(t) = -k(\xi(t) - a) - cv(t),$$

$$c_{33}(t) = -c/M,$$

$$c_{34}(t) = -c(\xi(t) - a)/M,$$

$$c_{35}(t) = -k/M,$$

$$c_{36}(t) = -c/M,$$

$$c_{41}(t) = -k(\xi(t) - a)/J,$$

$$c_{42}(t) = -[k(\xi(t) - a)^2 + cv(t)\,(\xi(t) - a)]/J,$$

$$c_{43}(t) = -c(\xi(t) - a)/J,$$

$$c_{44}(t) = -c(\xi(t) - a)^2/J,$$

$$c_{45}(t) = -k(\xi(t) - a)/J,$$

$$c_{46}(t) = -c(\xi(t) - a)/J,$$

$$c_{61}(t) = -k/m,$$

$$c_{62}(t) = -[k(\xi(t) - a) + cv(t)]/m,$$

$$c_{63}(t) = -c/m,$$

$$c_{64}(t) = -c(\xi(t) - a)/m$$

and $c_{ij}(t) = 0$ for all other indices, and

$$\gamma(t) = \begin{bmatrix} 0 \\ 0 \\ (Mg + k\eta_0 + \alpha_1^k z_{1,0} + \alpha_2^k z_{2,0})/M \\ (k(\xi(t) - a) - a\alpha_1^k z_{1,0} + b\alpha_2^k z_{2,0})/J \\ 0 \\ (k\eta_0 - mg)/m \end{bmatrix}.$$

Since the uncertainties in $m,\ k,\ c,\ \xi(t)$ and $v(t)$ are considered to be bounded, there exist some constants $\varepsilon_0 > 0$ and $\varepsilon_1 > 0$ such that

(i) $\|\gamma(t)\| \leqslant \varepsilon_0$,
(ii) $\|\mathbf{C}(t)\| \leqslant \varepsilon_1$.

This leads us to

$$\begin{aligned} \|\hat{\mathbf{e}}(\hat{\mathbf{x}}, t)\| &= \|\mathbf{C}(t)\hat{\mathbf{x}} + \gamma(t)\| \\ &\leqslant \|\mathbf{C}(t)\|\,\|\hat{\mathbf{x}}\| + \|\gamma(t)\| \\ &\leqslant \varepsilon_1\|\hat{\mathbf{x}}\| + \varepsilon_0. \end{aligned}$$

The control vector $\mathbf{u}$ is based on the measured state variables $\mathbf{x}$ and is given by

$$\mathbf{u} = \begin{bmatrix} p_1(\mathbf{x}) \\ p_2(\mathbf{x}) \end{bmatrix},$$

$$\begin{aligned} p_j(x) &= -\operatorname{sgn}(\mathbf{x}^{\mathrm{T}}\mathbf{PB}(\mathbf{x})\mathbf{i}_j) \\ &= -\operatorname{sgn}(b_j(\mathbf{x})) \end{aligned}$$

for $j = 1, 2$. $\hat{\mathbf{A}}$ is a stable matrix for any arbitrary but fixed and bounded uncertainties in $m,\ k$ or c in $\hat{\mathbf{A}}$. Hence, there exists a positive definite $\hat{\mathbf{P}} \in \mathbb{R}^{6,6}$ for any given positive definite $\hat{\mathbf{Q}} \in \mathbb{R}^{6,6}$ such that

$$\hat{\mathbf{Q}} = -(\hat{\mathbf{P}}\hat{\mathbf{A}} + \hat{\mathbf{A}}^{\mathrm{T}}\hat{\mathbf{P}})$$

holds. Now we consider the modified Lyapunov derivative for the state variables $\hat{\mathbf{x}}$ and an arbitrary control $\mathbf{u}$ constrained by $\pm 1 (j = 1, 2)$. In addition — since $\mathbf{B}(\mathbf{x})$ is linear in $\mathbf{x}$ — we use the property

$$\mathbf{B}(\mathbf{x})\mathbf{u} = \left(\sum_{j=1}^{2} \mathbf{B}_j u_j\right)\mathbf{x}$$

for some appropriate constant matrices $\mathbf{B}_j \in \mathbb{R}^{4,4}$ $(j = 1, 2)$. Then

$$\begin{aligned}
\mathscr{L}_{(\hat{\mathbf{x}},t)}(\mathbf{u}) &= -\tfrac{1}{2}\hat{\mathbf{x}}^{\mathrm{T}}\hat{\mathbf{Q}}\hat{\mathbf{x}} - \hat{\mathbf{x}}^{\mathrm{T}}\hat{\mathbf{P}}\begin{bmatrix}\mathbf{B}(\mathbf{x})\mathbf{u}\\ \mathbf{0}\end{bmatrix} + \hat{\mathbf{x}}^{\mathrm{T}}\hat{\mathbf{P}}\hat{\mathbf{e}}(\hat{\mathbf{x}}, t)\\
&\leqslant -\tfrac{1}{2}\lambda_{\min}(\hat{\mathbf{Q}})\|\hat{\mathbf{x}}\|^2 + \|\hat{\mathbf{x}}\|\,\|\hat{\mathbf{P}}\|\,\|\mathbf{B}(\mathbf{x})\cdot\mathbf{u}\| + \|\hat{\mathbf{x}}\|\,\|\hat{\mathbf{P}}\|\,\|\hat{\mathbf{e}}(\hat{\mathbf{x}}, t)\|\\
&\leqslant -\tfrac{1}{2}\lambda_{\min}(\hat{\mathbf{Q}})\|\hat{\mathbf{x}}\|^2 + \|(\hat{\mathbf{P}})\|\Big(\Big\|\Big(\textstyle\sum_{j=1}^{2}\mathbf{B}_j u_j\Big)\mathbf{x}\Big\| + \varepsilon_1\|\hat{\mathbf{x}}\| + \varepsilon_0\Big)\|\hat{\mathbf{x}}\|\\
&\leqslant -\tfrac{1}{2}\lambda_{\min}(\hat{\mathbf{Q}})\|\hat{\mathbf{x}}\|^2 + \lambda_{\max}(\hat{\mathbf{P}})\Big(\textstyle\sum_{j=1}^{2}\|\mathbf{B}_j\|\cdot|u_j|\,\|\mathbf{x}\| + \varepsilon_1\|\mathbf{x}\| + \varepsilon_0\Big)\|\hat{\mathbf{x}}\|\\
&\leqslant -\tfrac{1}{2}\lambda_{\min}(\hat{\mathbf{Q}})\|\hat{\mathbf{x}}\|^2 + \lambda_{\max}(\hat{\mathbf{P}})\Big(\textstyle\sum_{j=1}^{2}\|\mathbf{B}_j\|\cdot\|\mathbf{x}\| + \varepsilon_1\|\mathbf{x}\| + \varepsilon_0\Big)\|\hat{\mathbf{x}}\|\\
&\leqslant \Big(\lambda_{\max}(\hat{\mathbf{P}})\cdot\Big[\varepsilon_1 + \textstyle\sum_{j=1}^{2}\|\mathbf{B}_j\|\Big] - \tfrac{1}{2}\lambda_{\min}(\hat{\mathbf{Q}})\Big)\|\hat{\mathbf{x}}\|^2 + \varepsilon_0\lambda_{\max}(\hat{\mathbf{P}})\|\hat{\mathbf{x}}\|.
\end{aligned}$$

Thus, we obtain

$$\mathscr{L}_{(\hat{\mathbf{x}},t)}(\mathbf{u}) < 0 \quad \forall\|\hat{\mathbf{x}}\| > \hat{\rho} := \frac{2\varepsilon_0\lambda_{\max}(\hat{\mathbf{P}})}{\lambda_{\min}(\hat{\mathbf{Q}}) - 2\lambda_{\max}(\hat{\mathbf{P}})[\varepsilon_1 + \sum_{j=1}^{2}\|\mathbf{B}_j\|]}.$$

Consequently, provided

$$\lambda_{\min}(\hat{\mathbf{Q}}) > 2\lambda_{\max}(\hat{\mathbf{P}})\left[\varepsilon_1 + \sum_{j=1}^{2}\|\mathbf{B}_j\|\right]$$

holds, a ball of ultimate boundedness is given by

$$\mathscr{B}_{\hat{\rho}} = \{\hat{\mathbf{x}} \in \mathbb{R}^6 \mid \|\hat{\mathbf{x}}\| \leqslant \hat{\rho}\}.$$

Compared to Example 4.1, the excitation (that is the crossing truck) acts on the bridge within a finite time interval, say for $t \in [0, T]$ where $T > 0$. At $t = T$, the truck leaves the bridge and the bridge will oscillate freely (unforced and damped).

Therefore, it seems to make sense to record $z(t)$, $\Theta(t)$ and $\eta(t)$ for a certain time interval $[0, t_1]$ where t_1 is somewhat larger than T say, for instance, $t_1 = 2T$. The controller is based on the subsystem (bridge)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u} + \mathbf{e}(\mathbf{x}, \mathbf{y}, t)$$

with

$$\mathbf{A} = (a_{ij})_{\substack{i=1,\dots,4\\ j=1,\dots,4}}$$

where $a_{ij} = \hat{a}_{ij}$ as given above, and

$$\mathbf{I} = -(\mathbf{P}\mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{P}),$$

$$\mathbf{e}(\mathbf{x}, \mathbf{y}, t) = \begin{bmatrix}\hat{\mathbf{e}}_1(\hat{\mathbf{x}}, t)\\ \vdots\\ \hat{\mathbf{e}}_4(\hat{\mathbf{x}}, t)\end{bmatrix}.$$

The simulation result is based on the following realisation:

Truck: $m = 10^4$ [kg]
$k = 4 \times 10^5$ [N/m]
$c = 10^4$ [Ns/m]
$\eta_0 = 1$ [m]
$v(t) = 30$ km/h (= 8.33 [m/s])
$\xi(t) = 8.33$ [m/s] t
$T = 6$ [s]

Bridge: $M = 10^5$ [kg]
$J = 2 \times 10^7$ [kgm^2]
$a = b = 25$ [m]
$\alpha_i^k = 4 \times 10^6$ [N/m] $(i = 1,2)$
$\beta_i^k = 10^6$ [N/m]) $(i = 1,2)$
$\alpha_i^c = 5 \times 10^4$ [Ns/m] $(i = 1,2)$
$\beta_i^c = 10^4$ [Ns/m] $(i = 1,2)$.

We choose $z_{1,0}$ and $z_{2,0}$ in such a way that the bridge without truck is in an equilibrium position at $z = 0$ if no control ($u_j = 0$) is applied. That is

$$Mg + \alpha_1^k z_{1,0} + \alpha_2^k z_{2,0} \doteq 0,$$

$$b\alpha_2^k z_{2,0} - a\alpha_1^k z_{1,0} \doteq 0.$$

Then, because of $a = b$ and $\alpha_1^k = \alpha_2^k$

$$z_{1,0} = z_{2,0} = -\frac{Mg}{2\alpha_1^k} = -0.125 \text{ [m]}.$$

Fig. 12 shows the vertical deviation $z(t)$ for the controlled and uncontrolled bridge. After $T = 6$ s the bridge carries out a vibratory motion without truck. Fig. 13 shows the angular declination $\Theta(t)$ for the controlled and uncontrolled case. As it can be easily seen, the bridge changes the inclination linearly while the truck crosses with constant speed. After the truck leaves the bridge, the inclination Θ jumps back to

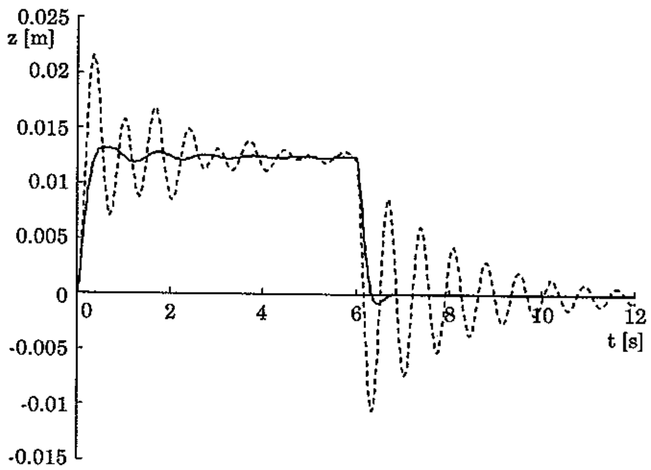

Fig. 12. $z(t)$ controlled (—) and uncontrolled (- - -).

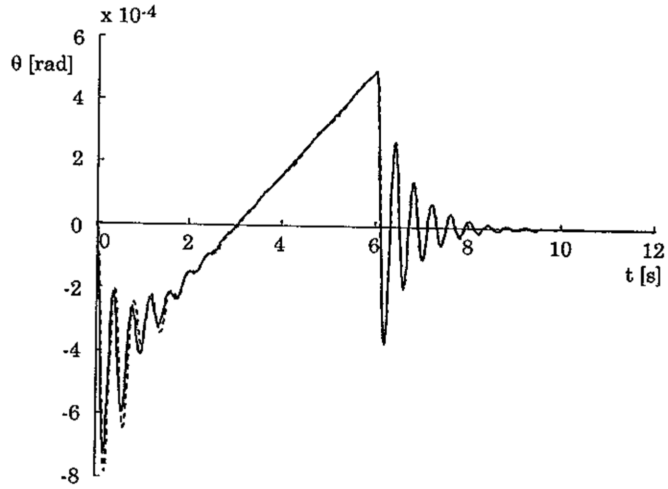

Fig. 13. $\Theta(t)$ controlled (—) and uncontrolled (- - -).

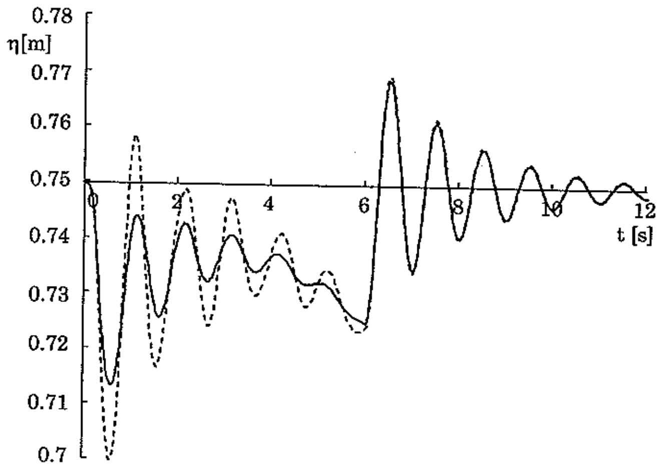

Fig. 14. $\eta(t)$ for $0 \leqslant t \leqslant 2T$ (bridge controlled (—) and bridge uncontrolled (- - -)).

zero in a damped oscillatory motion. Fig. 14 shows in addition the free oscillation of the truck. Its vibration is not assumed to be controlled.

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