ADVANCES IN STRUCTURAL CONTROL
J. Rodellar, A.H. Barbat and F. Casciati (editors)

Chapter 5

Robust Control of Seismic Structures Employing Active Suspension Elements

Eduard Reithmeier
Institut für Meß- und Regelungstechnik
Universität Hannover
Germany

George Leitmann
College of Engineering
University of California
Berkeley, California, USA

5.1 Introduction

Undesired time-variant displacements of seismic structures (i.e. in scales, balances, vibratory platforms or even buildings) are mainly caused by unknown or uncertain excitations. In a variety of applications it is desirable or even necessary to attenuate these disturbances in an effective way and with moderate effort. Hence, several passive as well as active methods and techniques have been developed in order to treat these problems. However, employment of active techniques often fails because of their considerable financial costs.

We propose an affordable semiactive control scheme which accounts for the above mentioned deficiencies. In addition, we allow constraints on control actions. Furthermore, the number of control inputs (actuators) may be arbitrary, that is, the system may be mismatched. The scheme is based on Lyapunov stability theory and, provided that the bounds of the uncertainties are also a priori known, a stable attractor (*ball of ultimate boundedness*) of the structure can be computed (Leitmann, 1979; Corless and Leitmann, 1981). In case measurement errors or uncertainties, respectively, are significant, it is shown how the Lyapunov based control scheme may be combined with a fuzzy control concept.

The class of structures which we will take into consideration may be composed of "rigid" parts as well as "active" and "passive" elastic coupling elements. We call suspension or coupling elements "active" if they are adjustable with respect to their stiffness and damping behavior. Based on that model we assume that a control action is related to a change in these properties. The mathematical description of these kinds of structures is assumed to be an initial value problem of the form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{u}) + \boldsymbol{e}(\boldsymbol{x},\boldsymbol{z},t) \quad , \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \ . \tag{5.1}$$

The linear part of the mathematical model of the structure is defined by the constant and stable matrix $\boldsymbol{A} \in \mathbb{R}^{n,n}$, where $n \in \mathbb{N}$ denotes the state space dimension. The control input function is assumed to be continous in $\boldsymbol{x}, \boldsymbol{z}$ and $\boldsymbol{u}$, and linear in $\boldsymbol{u}$, that is

$$\boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{u}) \in \mathbb{R}^n \quad \text{with} \quad \boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\alpha\boldsymbol{v}+\beta\boldsymbol{w}) = \alpha\boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{v})+\beta\boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) \tag{5.2}$$

for all $\boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{z} \in \mathbb{R}^r$, $\boldsymbol{v}, \boldsymbol{w} \in \mathbb{R}^m$ and $\alpha, \beta \in \mathbb{R}$. $\boldsymbol{x} \in \mathbb{R}^n$ represents the n state variables; $\boldsymbol{z} \in \mathbb{R}^r$ the r "disturbances", whose current values can be measured, $\boldsymbol{u} \in \mathbb{R}^m$ the m control variables. Furthermore, we will assume that the m vectors $\boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\mathbf{i}_j)$, $(j = 1,\ldots,m)$ are linearly independent for all $\boldsymbol{x} \in \mathbb{R}^n$ and $\boldsymbol{z} \in \mathbb{R}^r$:

$$\Rightarrow \quad rk[\boldsymbol{B}(\boldsymbol{x},\boldsymbol{z})] = m \quad \forall \quad (\boldsymbol{x},\boldsymbol{z}) \in \mathbb{R}^n \times \mathbb{R}^r \tag{5.3}$$

where $\mathbf{i}_j$ is the unit vector in the j-th direction, $\mathbf{i}_i^T\mathbf{i}_j = 0$ for $i \neq j$, that is $\boldsymbol{u} = \sum_{j=1}^m u_j\mathbf{i}_j$ and

$$\boldsymbol{B}(\boldsymbol{x},\boldsymbol{z}) := [\boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\mathbf{i}_1),\ldots,\boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\mathbf{i}_m)] \in \mathbb{R}^{n,m} \ . \tag{5.4}$$

That is, we assume that the control variables are not redundant or, in other words, every control variable acts in a different direction. The control variables are supposed to be constrained by lower and upper bounds:

$$\mathcal{U} := [u_{1,min}, u_{1,max}] \times \ldots \times [u_{m,min}, u_{m,max}] \ . \tag{5.5}$$

This restriction reflects the fact that applied control actions in practice are restricted because of technical reasons. Without loss of generality and for the sake of convenience, we may assume that

$$\mathcal{U} = [-1, 1] \times \ldots \times [-1, 1] \ . \tag{5.6}$$

All unknown or uncertain "excitations/effects" are modelled by

$$e : \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R} \to \mathcal{B}_\eta := \{\xi \in \mathbb{R}^n \mid \|\xi\| \le \eta \in \mathbb{R}_+\} \ . \tag{5.7}$$

That is, all uncertainties are assumed to be bounded by the compact ball $\mathcal{B}_\eta$ with radius η:

$$\|e(x, z, t)\| \le \eta \in \mathbb{R}_+ \quad \forall \quad (x, z, t) \in \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R}_+ \ . \tag{5.8}$$

We assume also e is at least piecewise continuous. Furthermore, we assume that the state x as well as the disturbances z are available via some measurement devices. In case of seismic disturbances, z may be determined by seismograph records. That is, x and z are considered as plant output $y := (x, z)$ (see Figure 5.1).

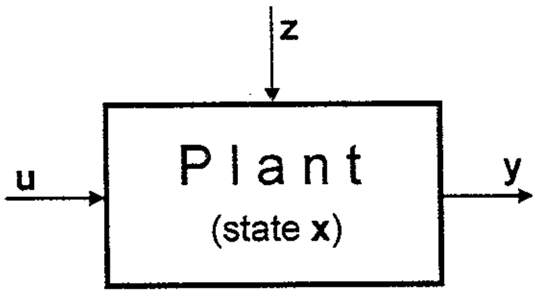

Figure 5.1: Input and output variables of the plant.

5.2 Control design

5.2.1 Lyapunov approach

Following the controller design in (Leitmann *et al.*, 1993; Leitmann, 1994) we ask for a feedback

$$p : \mathbb{R}^n \times \mathbb{R}^r \to \mathcal{U} \ , \quad (x, z) \mapsto u = p(x, z)) \tag{5.9}$$

such that, for any given positive definite matrix $P \in \mathbb{R}^{n,n}$, the time derivative of

$$\|x(t)\|_P := \sqrt{x^T(t)Px(t)} \quad , \quad t \in \mathbb{R}_+ \tag{5.110}$$

is as small as possible for any:

(i) response $t \mapsto x(t)$,
(ii) disturbance $z(t)$,
(iii) admissible uncertainty $e(x(t), z(t), t)$,
(iv) and time t ,

and for all admissible choices of control $u(t)$. This means that, based on the Lyapunov function candidate

$$V(x) := \frac{1}{2} \|x\|_P^2 \ . \tag{5.111}$$

the feedback p we are seeking is one which minimizes the Lyapunov derivative

$$\begin{aligned} \mathcal{L}_{(x,z,t)}(u) &:= x^T P [Ax + b(x, z, u) + e(x, z, t)] \\ &= x^T P \left[Ax + \sum_{j=1}^{m} u_j b(x, z, \mathbf{i}_j) + e(x, z, t)\right] \end{aligned} \tag{5.112}$$

with respect to u for every $(x, z, t) \in \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R}_+$. In other words,

$$\mathcal{L}_{(x,z,t)}(p(x, z)) = \min \left\{L_{(x,z,t)}(u) \mid u \in \mathcal{U}\right\} \ . \tag{5.113}$$

Equation (5.12) can be written as

$$\mathcal{L}_{(x,z,t)}(u) = a(x) + \sum_{j=1}^{m} u_j b_j(x, z) + c(x, z, t) \tag{5.114}$$

with

$$\begin{aligned} a(x) &:= -\tfrac{1}{2} x^T Q x \quad &\text{where} \quad Q &:= -(PA + A^T P) \ , \\ b_j(x, z) &:= x^T P b(x, z, \mathbf{i}_j) \quad &\text{where} \quad \mathbf{i}_j \mathbf{i}_k &= \delta_{jk} \end{aligned} \tag{5.115}$$

and

$$c(x, z, t) := x^T P e(x, z, t) \ . \tag{5.116}$$

Then

$$u_j = p_j(x, z) = \begin{cases} u_{j,min} & \text{if} \quad b_j(x, z) > 0 \\ u_{j,max} & \text{if} \quad b_j(x, z) < 0 \end{cases} \qquad (j = 1, \ldots, m) \quad (5.17)$$

Using the normalized control space $\mathcal{U}$ defined by equation (5.6) we obtain

$$u_j = p_j(x, z) = -\text{sgn}[b_j(x, z)]. \qquad (5.18)$$

The performance of the controller may be enhanced additionally if we choose $\boldsymbol{P}$ appropriately. The smaller the Lyapunov derivative, the stronger the "tendency to the origin" of $t \mapsto x(t)$. Our objective in that case would be to strive for a highly negative value $a(x)$ in equation (5.15). That, on the other hand, can be done by choosing a suitable positive definite $\boldsymbol{Q}$ and solving the algebraic Lyapunov equation

$$\boldsymbol{Q} = -(\boldsymbol{PA} + \boldsymbol{A}^T\boldsymbol{P}) \qquad (5.19)$$

for $\boldsymbol{P}$. Since $\boldsymbol{A}$ is assumed to be stable, the matrix $\boldsymbol{P}$ is positive definite.

5.2.2 Stability

Using the control scheme developed in Section 5.2.1 it is possible to determine a region (compact set) toward which any realization $t \mapsto x(t)$ is attracted and wherein it remains once it has crossed the boundary of the "attractor". In order to analyze this situation we employ the controller $\boldsymbol{p}$ (cf. equation (5.18)) in expression (5.14). This leads to

$$\mathcal{L}_{(\boldsymbol{x},\boldsymbol{z},t)}(\boldsymbol{p}(\boldsymbol{x}, \boldsymbol{z})) = -\frac{1}{2}\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} - \sum_{j=1}^{m} |b_j(\boldsymbol{x}, \boldsymbol{z})| + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{e}(\boldsymbol{x}, \boldsymbol{z}, t) \ , \quad (5.20)$$

where $\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x}$ is a positive definte quadratic form. Hence, the form is bounded by the minimum and maximum eigenvalue λ_{min} and λ_{max} of $\boldsymbol{Q}$. That is,

$$\mathcal{L}_{(\boldsymbol{x},\boldsymbol{z},t)}(\boldsymbol{p}(\boldsymbol{x}, \boldsymbol{z})) \leq -\frac{1}{2}\lambda_{min}(\boldsymbol{Q})\|\boldsymbol{x}\|^2 - \sum_{j=1}^{m} |b_j(\boldsymbol{x}, \boldsymbol{z})| + \|\boldsymbol{x}\|\|\boldsymbol{P}\|\|\boldsymbol{e}(\boldsymbol{x}, \boldsymbol{z}, t)\| \quad (5.21)$$

or, neglecting the second term, employing inequality (5.8) and using the upper bound $\lambda_{max}(\boldsymbol{P})$ of $||\boldsymbol{P}||$

$$\mathcal{L}_{(\boldsymbol{x},\boldsymbol{z},t)}(\boldsymbol{p}(\boldsymbol{x},\boldsymbol{z})) \leq -\frac{1}{2}\lambda_{min}(\boldsymbol{Q})||\boldsymbol{x}||^2 + \lambda_{max}(\boldsymbol{P})\eta||\boldsymbol{x}|| \ . \tag{5.22}$$

Thus, we obtain

$$\mathcal{L}_{(\boldsymbol{x},\boldsymbol{z},t)}(\boldsymbol{p}(\boldsymbol{x},\boldsymbol{z})) \leq 0 \qquad \forall \qquad ||\boldsymbol{x}|| > r \tag{5.23}$$

where

$$r := \frac{2 \cdot \eta \cdot \lambda_{max}(\boldsymbol{P})}{\lambda_{min}(\boldsymbol{Q})} \ . \tag{5.24}$$

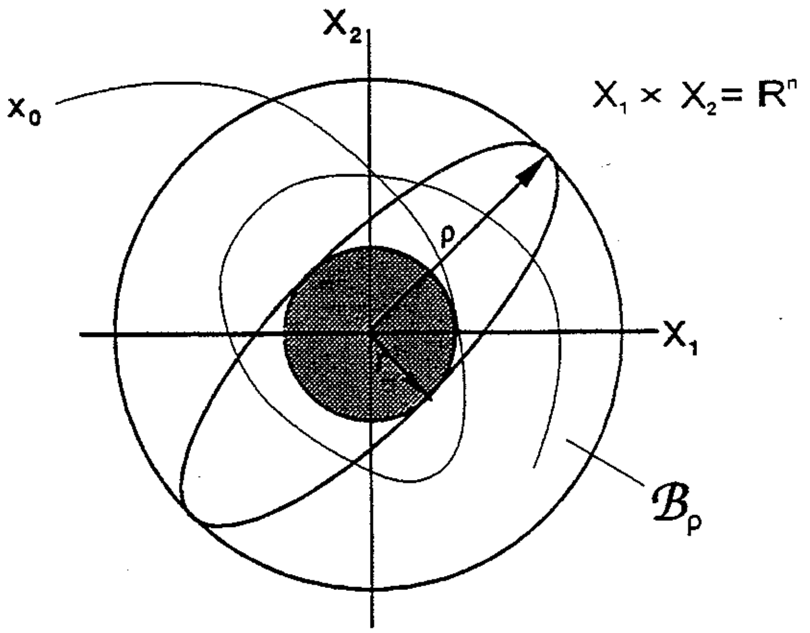

Figure 5.2: Ball of ultimate boundedness.

The radius ρ of the *ball of ultimate boundedness* (cf. Figure 5.2) is therefore determined by

$$\rho = r \cdot \sqrt{\frac{\lambda_{max}(\boldsymbol{P})}{\lambda_{min}(\boldsymbol{P})}} \ , \tag{5.25}$$

and any response $t \mapsto \boldsymbol{x}(t)$ which enters

$$\mathcal{B}_\rho := \{\boldsymbol{\xi} \in \mathbb{R}^n \mid ||\boldsymbol{\xi}|| \leq \rho\} \tag{5.26}$$

say at $t = t^*$ remains in $\mathcal{B}_\rho$ for all $t > t^*$. It should be noted that $\mathcal{B}_\rho$, with radius given in (5.25), is the ball of ultimate boundedness for the uncontrolled system, since the terms $\sum_{j=1}^{m} |b_j(x,z)|$ were neglected in (5.21). These terms reduce the right–hand side of (5.21), except at (x,z) where the $b_j(x,z) = 0$ for all j. In other words, the control acts to reduce the radius of $\mathcal{B}_\rho$ as well as the rate of convergence.

5.3 Measurement errors

5.3.1 General aspects

A change in the control variables u_j takes place only if the *indicator function* b_j, defined in (5.15) changes its sign. That is, a change in u_j occurs if a response $t \mapsto y(t) = (x(t), z(t))$ crosses

$$\Pi_j := \{(x,z) \in \mathbb{R}^n \times \mathbb{R}^r \quad | \quad b_j(x,z) = 0\}. \tag{5.27}$$

We assume that Π_j denotes a differentiable manifold of dimension $n+r-1$; otherwise the space $\mathbb{R}^n \times \mathbb{R}^m$ would not be separated by Π_j into two disjoint parts. Furthermore, then

$$g_j : \Pi_j \to \mathbb{R}^n \times \mathbb{R}^r \quad , \quad \xi \mapsto y := g_j(\xi) \tag{5.28}$$

defines a unique and differentiable embedding of Π_j into $\mathbb{R}^n \times \mathbb{R}^r$ (cf. Figure 5.3).

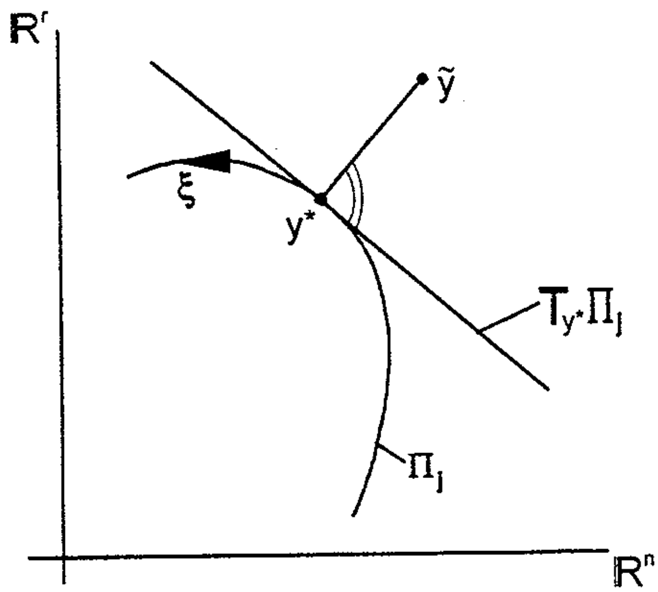

Figure 5.3: Embedding of Π_j in $\mathbb{R}^n \times \mathbb{R}^r$.

5.3.2 Fuzzy concept

Depending on the accuracy of the measurement devices there is always uncertainty concerning the measured variables. The assumed maximum difference between actual value y and measured value $\tilde{y}$ may be expressed by $\Delta y := (\Delta x_1, \ldots, \Delta x_n, \Delta z_1, \ldots, \Delta z_r)^T$, such that

$$\|\Delta y\|_M = 1 \ . \tag{5.29}$$

$\| \cdot \|_M : \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}_+$ is some norm which takes care of the tolerances and scales in measurement for the different variables. For instance, it might be defined by

$$\|y\|_M := \sqrt{\left(\frac{1}{n+r}\right)\left[\sum_{i=1}^{n}\left(\frac{x_i}{\Delta x_i}\right)^2 + \sum_{i=1}^{r}\left(\frac{z_i}{\Delta z_i}\right)^2\right]} \tag{5.30}$$

As mentioned before, a change in control u_j will occur only on the switching plane Π_j for $j = 1, \ldots, m$. Hence, we define uncertain transition regions $\mathcal{T}_j$ (cf. Figure 5.4) as follows: if the measured value $\tilde{y}$ indicates switching, then the actual value y belongs to the transition area, that is

$$\tilde{y} \in \Pi_j \quad \Longrightarrow \quad y \in \mathcal{T}_j \ . \tag{5.31}$$

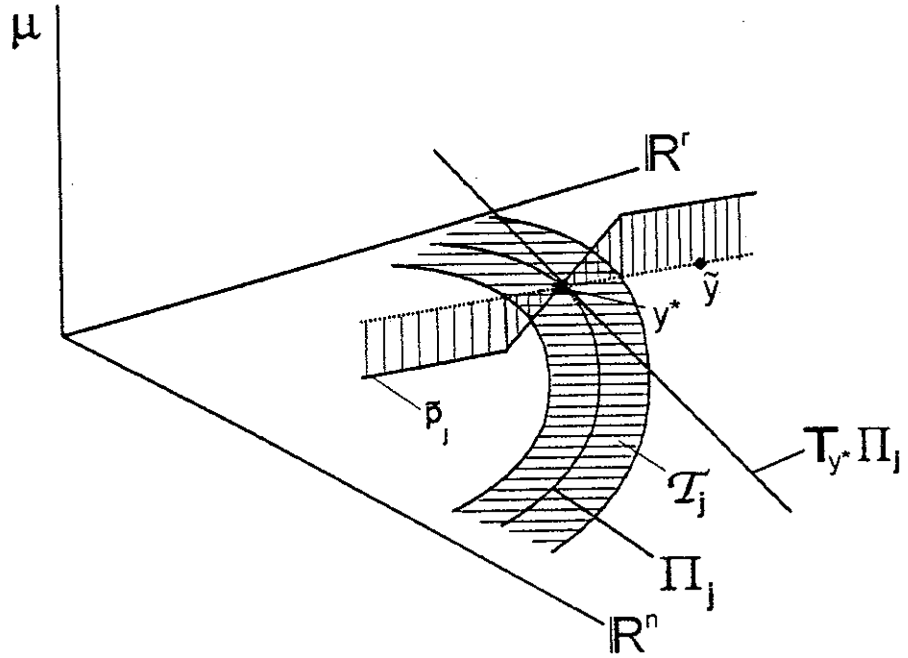

Figure 5.4: Uncertain transition area $\mathcal{T}_j$ and membership function μ_j.

The distance between $\tilde{y}$ and Π_j is assumed to be the perpendicular to the tangent plane $T_{y^*}\Pi_j$ at an appropriate point $y^* \in \Pi_j$ (cf. Figure 5.3). According to that definition. y^* is determined by solving the following algebraic equation:

$$[Dg_j(\xi^*)]^T(\tilde{y} - g_j(\xi^*)) = 0 \quad \Rightarrow \quad \xi^*(\tilde{y}) \quad \Rightarrow \quad y^* = g_j(\xi^*(\tilde{y})) \qquad (5.32)$$

Using equation (5.31) we are able to define $\mathcal{T}_j$ via

$$\mathcal{T}_j := \{\hat{y} \in \mathbb{R}^n \times \mathbb{R}^r \quad | \quad \|\hat{y} - g_j(\xi^*(\hat{y})\|_M \leq 1\} \qquad (5.33)$$

In order to replace the designed Lyapunov controller on $\mathcal{T}_j$ by some appropriate fuzzy controller we use relationship (5.32) as a *fuzzyfication* process. Within $\mathcal{T}_j$ the choice of the control variables may be right or may be wrong. If the decision is right the choice will be the best case. if the decision is wrong the choice will be the worst case. Therefore it seems very reasonable to *defuzzify* the transition area in such a way that there exists a linear relationship (membership function) within $\mathcal{T}_j$. Or. in terms of probability, we assume a linear distribution across $\mathcal{T}_j$, which says that the chance for $\tilde{y}$ to be right or to be wrong is 50% on the switching surfaces and 100% on the border of $\mathcal{T}_j$. In between it is supposed to be linearly increasing or decreasing, respectively (cf. Figure 5.4). Hence. the membership function is defined by

$$\mu : \mathcal{T}_j \to \mathcal{U} \quad , \quad \tilde{y} \mapsto \mu(\tilde{y}) \qquad (5.34)$$

with

$$\mu_j(\tilde{y}) := \begin{cases} 0 & \text{if} \quad \tilde{y} \in \Pi_j \\ \|\tilde{y} - y^*\|_M \cdot \operatorname{sgn}[b_j(\tilde{y})] & \text{if} \quad \tilde{y} \in \Pi_j - \mathcal{T}_j \end{cases} \qquad (j = 1, \ldots, m) \qquad (5.35)$$

Of course, for each j there exists a different y^* which needs to be determined via equation (5.32). The modified continuous state feedback is then given by

$$\tilde{p}_j(\tilde{y}) := \begin{cases} \mu_j(\tilde{y}) & \text{if} \quad \|\tilde{y} - y^*\|_M < 1 \\ \operatorname{sgn}[b_j(\tilde{y})] & \text{if} \quad \|\tilde{y} - y^*\|_M \geq 1 \end{cases} \qquad (5.36)$$

5.4 Example

An example of the class of systems modelled by (5.1) is a structure with two storeys as shown in Figure 5.5.

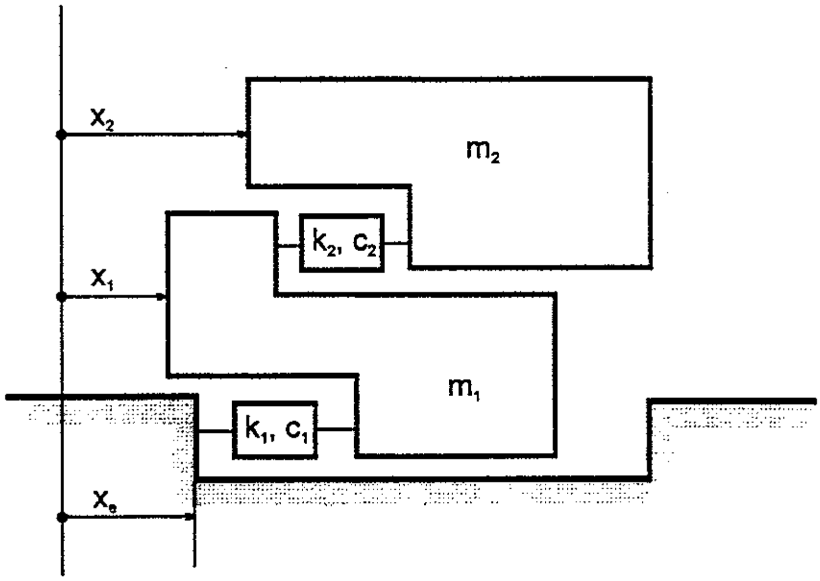

Figure 5.5: Structure with two storeys.

We will consider that example in order to demonstrate the efficacy and robustness of the proposed control scheme. In that case, the spring and damping coefficients k_i and c_i are linear functions of the applied control action v_i which, for instance, could be the voltage applied on suspension elements filled with so called smart materials:

$$\begin{aligned} k_j(v_j) &= \alpha_j^k + \beta_j^k v_j \qquad (j = 1, 2) \\ c_i(v_j) &= \alpha_j^c + \beta_j^c v_j \qquad (j = 1, 2) \end{aligned} \tag{5.37}$$

where $\alpha_j^k, \alpha_j^c, \beta_j^k$ and $\beta_j^c \in \mathbb{R}_+$ are constant parameters and the voltage v_j for spring/damper j can be varied between 0 and $\overline{v}_j > 0$. The linear parameter shift

$$u_j := \left[2 \cdot (\frac{v}{\overline{v}_j}) - 1\right] \tag{5.38}$$

transforms v_j into the normalized control variable u_j. The realization of the disturbance z considered here is a periodic ground displacement

$$t \mapsto x_e(t) := \hat{x}_e \cdot \sin[\nu t]. \tag{5.39}$$

The equation of motion is given by

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + [\boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\mathbf{i}_1)u_1 + \boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\mathbf{i}_2)u_2] + \boldsymbol{e}(\boldsymbol{x},\boldsymbol{z},t) \tag{5.40}$$

with

$$\boldsymbol{A} := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{1}{m_1}[\alpha_1^k + \alpha_2^k] & \frac{\alpha_2^k}{m_1} & -\frac{1}{m_1}[\alpha_1^c + \alpha_2^c] & \frac{\alpha_2^c}{m_1} \\ \frac{\alpha_2^k}{m_2} & -\frac{\alpha_2^k}{m_2} & \frac{\alpha_2^c}{m_2} & -\frac{\alpha_2^c}{m_2} \end{bmatrix}$$

and

$$\boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\mathbf{i}_1) := \begin{bmatrix} 0 \\ 0 \\ -\frac{\beta_1^k}{m_1}(x_1 - x_e) - \frac{\beta_1^c}{m_1}(x_3 - \dot{x}_e) \\ 0 \end{bmatrix},$$

$$\boldsymbol{b}(\boldsymbol{x},\boldsymbol{z},\mathbf{i}_2) := \begin{bmatrix} 0 \\ 0 \\ \frac{\beta_2^k}{m_1}(x_2 - x_1) + \frac{\beta_2^c}{m_1}(x_4 - x_3) \\ \frac{\beta_2^k}{m_2}(x_1 - x_2) + \frac{\beta_2^c}{m_2}(x_3 - x_4) \end{bmatrix},$$

$$\boldsymbol{e}(\boldsymbol{x},\boldsymbol{z},t) := \begin{bmatrix} 0 \\ 0 \\ \frac{\alpha_1^k}{m_1}x_e + \frac{\alpha_1^c}{m_1}\dot{x}_e \\ 0 \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} := \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix},$$

$$\boldsymbol{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} := \begin{bmatrix} x_e \\ \dot{x}_e \end{bmatrix}.$$

In earlier work, e.g. Kelly *et al.* (1987) or Leitmann (1994), it is assumed that $x_e, \dot{x}_e$ are not measured but that their bounds are known; in that event, x_e and $\dot{x}_e$ contributes to uncertainty in $\boldsymbol{e}(\boldsymbol{x}, \boldsymbol{z}, t)$.

The indicator function b_j may be expressed by

$$b_j(y) = \boldsymbol{x}^T \boldsymbol{P}\boldsymbol{B}_j \boldsymbol{y} \;, \quad (j = 1, 2), \tag{5.41}$$

where $\boldsymbol{B}_j \in \mathbb{R}^{n,n+r}$ are constant matrices. In addition

$$rk[\boldsymbol{B}_j] = 1 \tag{5.42}$$

holds for $j = 1$ and $j = 2$. Hence, an appropriate Householder transformation $\boldsymbol{H}_j \in \mathbb{R}^{n,n}$ leads to

$$\begin{aligned} b_j(\boldsymbol{y}) &:= (\boldsymbol{x}^T \boldsymbol{P}\boldsymbol{H}_j^T) \cdot (\boldsymbol{H}_j \boldsymbol{B}_j \boldsymbol{y}) \\ &= (\boldsymbol{x}^T \boldsymbol{P}\boldsymbol{H}_j^T e_1) \cdot (\boldsymbol{c}^T \boldsymbol{y}) \end{aligned} \tag{5.43}$$

where (i) $\boldsymbol{H}_j^{-1} = \boldsymbol{H}_j^T$,

(ii) $\boldsymbol{H}_j \boldsymbol{B}_j \boldsymbol{y} = (\boldsymbol{c}^T \boldsymbol{y})\mathrm{e}_1$,

(iii) $\boldsymbol{c} = const. \in \mathbb{R}^{n+r}$, $e_1 := (1, 0, \ldots, 0)^T \in \mathbb{R}^n$.

Taking this into consideration, we obtain finally the following statement:

$$b_j(\boldsymbol{y}) = 0 \quad \Longleftrightarrow \quad \boldsymbol{x}^T \boldsymbol{P}\boldsymbol{H}_j^T \mathrm{e}_1 = 0 \quad \text{and/or} \quad \boldsymbol{c}^T \boldsymbol{y} = 0. \tag{5.44}$$

That is, the manifold Π_j degenerates into two planes whose intersection contains the origin $\boldsymbol{0}$ (cf. Figure 5.6). That means, in particular, that

$$T\boldsymbol{y}\Pi_j = \Pi_j \quad \forall \quad \boldsymbol{y} \in \Pi_j. \tag{5.45}$$

The following simulations are for initial value problems with $\boldsymbol{x}_0 = \boldsymbol{0}$ (cf. Equation (5.1)). The dynamical system is integrated for $15\,s$ and an "amplitude" of each state component x_i is defined as the maximum of its absolute value during the final $5\,s$. All results are based on the parameter set ($i = 1, 2$):

$$\overline{v}_i = 10^3\,\mathrm{V}, \quad m_i = 10\,\mathrm{kg}, \quad \hat{x}_e = 0.02\,\mathrm{m}$$

and

$$\alpha_i^c = 4\,\frac{\text{Ns}}{\text{m}},\ \beta_i^c = 8\,\frac{\text{Ns}}{\text{Vm}},\ \alpha_i^k = 2\cdot 10^3\,\frac{\text{N}}{m},\ \beta_i^k = 10^3\,\frac{\text{N}}{\text{Vm}}.$$

The fuzzy controller is defined according to equation (5.36) with

$$\begin{aligned} \|y\|_M^2 &:= \left(\frac{1}{4+2}\right)\left[\left(\frac{x_1}{\Delta x_1}\right)^2 + \left(\frac{x_2}{\Delta x_2}\right)^2 + \left(\frac{\dot{x}_1}{\Delta \dot{x}_1}\right)^2 + \left(\frac{\dot{x}_2}{\Delta \dot{x}_2}\right)^2\right] \\ &+ \left(\frac{1}{4+2}\right)\left[\left(\frac{x_e}{\Delta x_e}\right)^2 + \left(\frac{\dot{x}_e}{\Delta \dot{x}_e}\right)^2\right] \end{aligned}$$

where Δx_1, Δx_2, $\Delta \dot{x}_1$, $\Delta \dot{x}_2$, Δx_e and $\Delta \dot{x}_e$ are the maximum errors which may occur during measurement. They are given by

$$\Delta x := \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 5\cdot 10^{-3} m \\ 5\cdot 10^{-3} m \\ 0.1 m/s \\ 0.1 m/s \end{bmatrix} \quad \text{and} \quad \Delta z := \begin{bmatrix} \Delta x_e \\ \Delta \dot{x}_e \end{bmatrix} = \begin{bmatrix} 5\cdot 10^{-3} m \\ 0.1 m/s \end{bmatrix}.$$

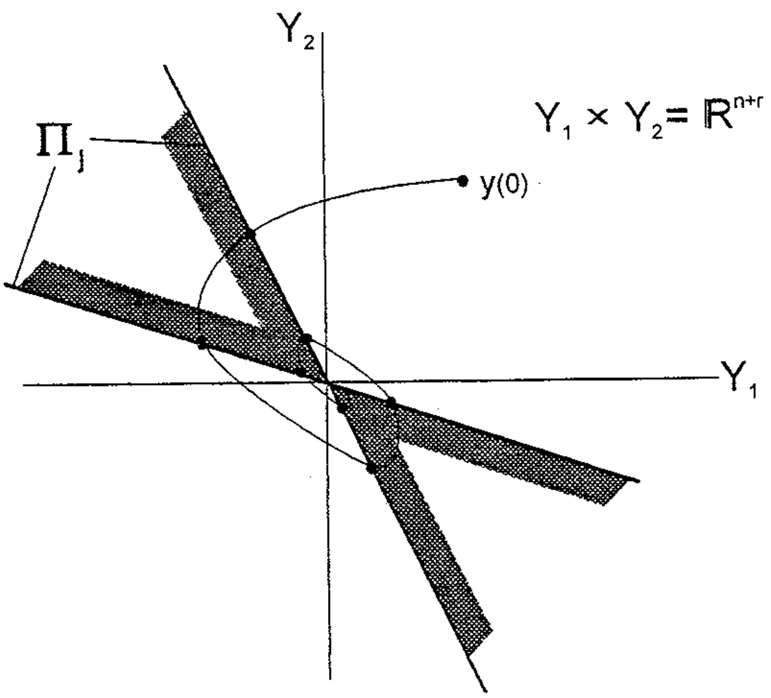

Figure 5.6: Switching plane Π_j for u_j.

The results are shown from Figure 5.7 to Figure 5.10. Each diagram shows the four different cases:

- Only the Lyapunov controller is used.
- The Lyapunov and Fuzzy controller are combined.
- Constant maximum damping and maximum stiffness is applied ($u_j = u_{j,max}$).
- No control at all is applied ($u_j = 0$).

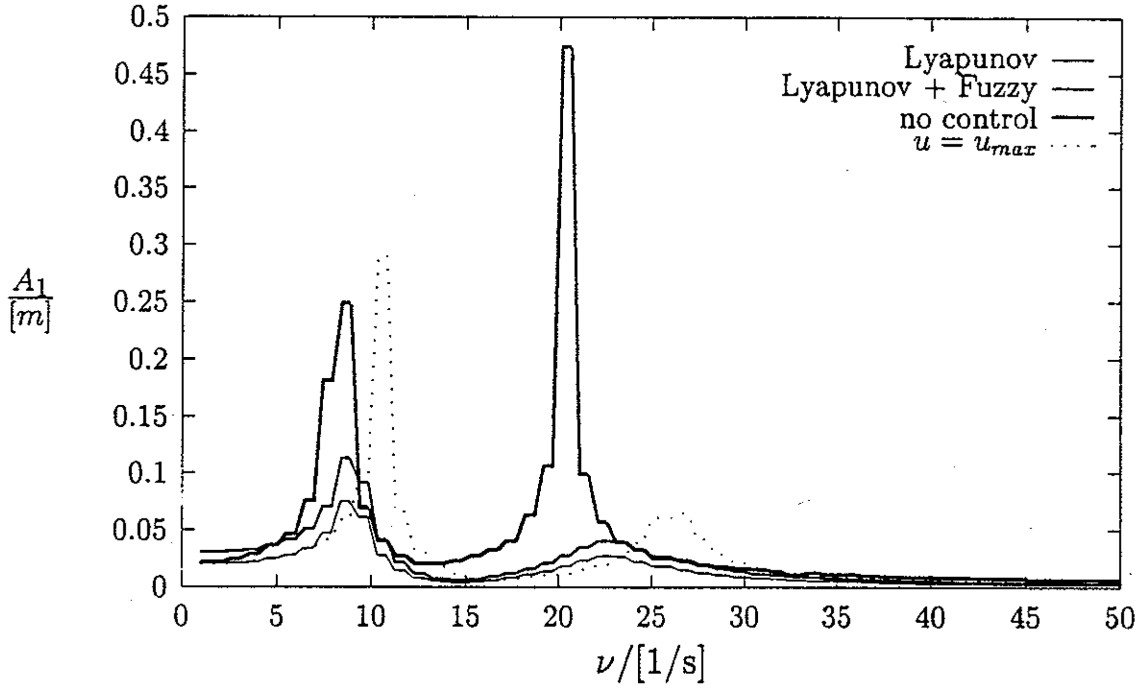

Figure 5.7: Maximum amplitude A_1 of x_1 versus excitation frequency ν.

As one can easily recognize, there is a significant suppression for all state components x_i in the controlled cases. The cases "constant minimum stiffness and damping" and "constant maximum stiffness and damping" are significantly worse near their resonance frequencies. Although, the "pure Lyapunov controller" leads to the best results, it does not account for measurement errors. Taking into consideration that assumed measurement errors may rise up to 25% of the ground diplacement x_e and up to 50% for low frequencies ν, the combined "Lyapunov and Fuzzy" approach seems to be a very reasonable choice.

Of course, the smaller the Euclidian norm $||\Delta y||$ of the maximum measurement errors Δy_i the smaller the transition areas $\mathcal{T}_j$ around the switching planes Π_j and the smaller the difference between the Lyapunov controller and the combined control "Lyapunov + Fuzzy".

It is important to note that the significant suppression of time variant displacements of the state components x_i takes place in resonant frequency

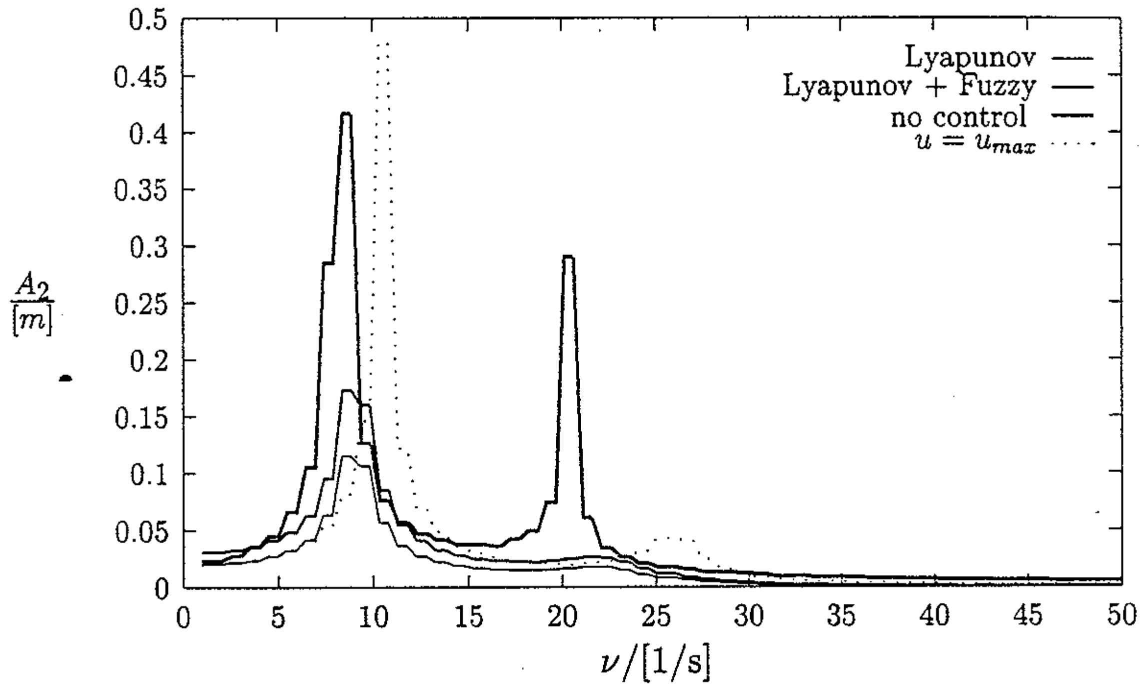

Figure 5.8: Maximum amplitude A_2 of x_2 versus excitation frequency ν.

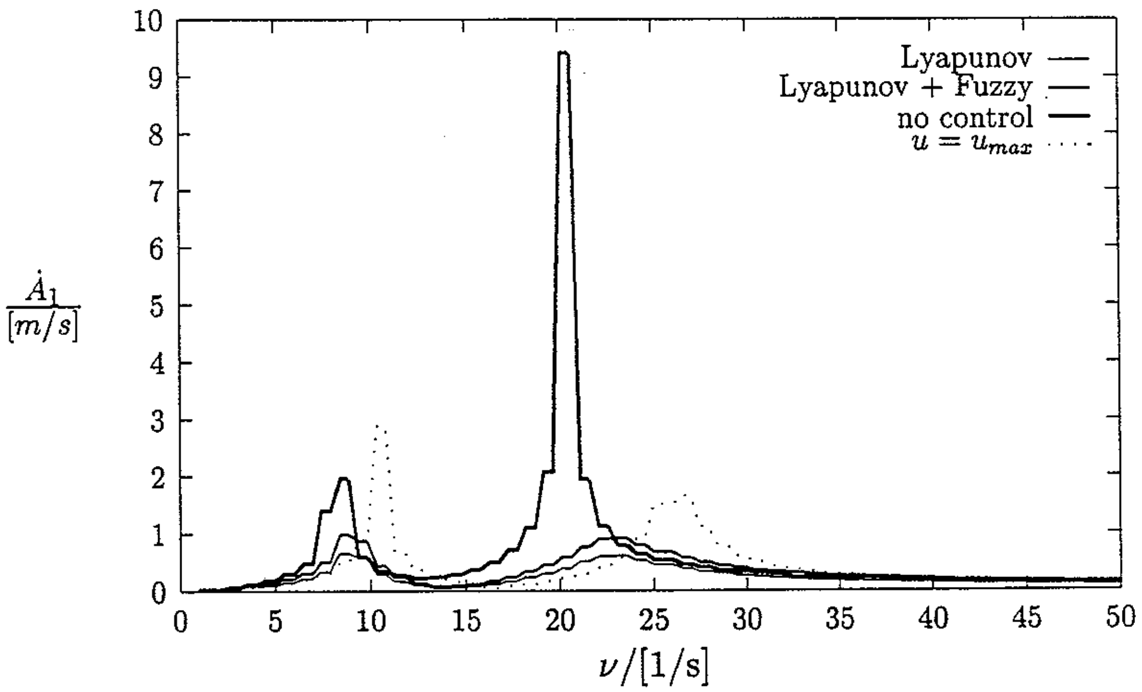

Figure 5.9: Maximum amplitude $\dot{A}_1$ of $\dot{x}_1$ versus excitation frequency ν.

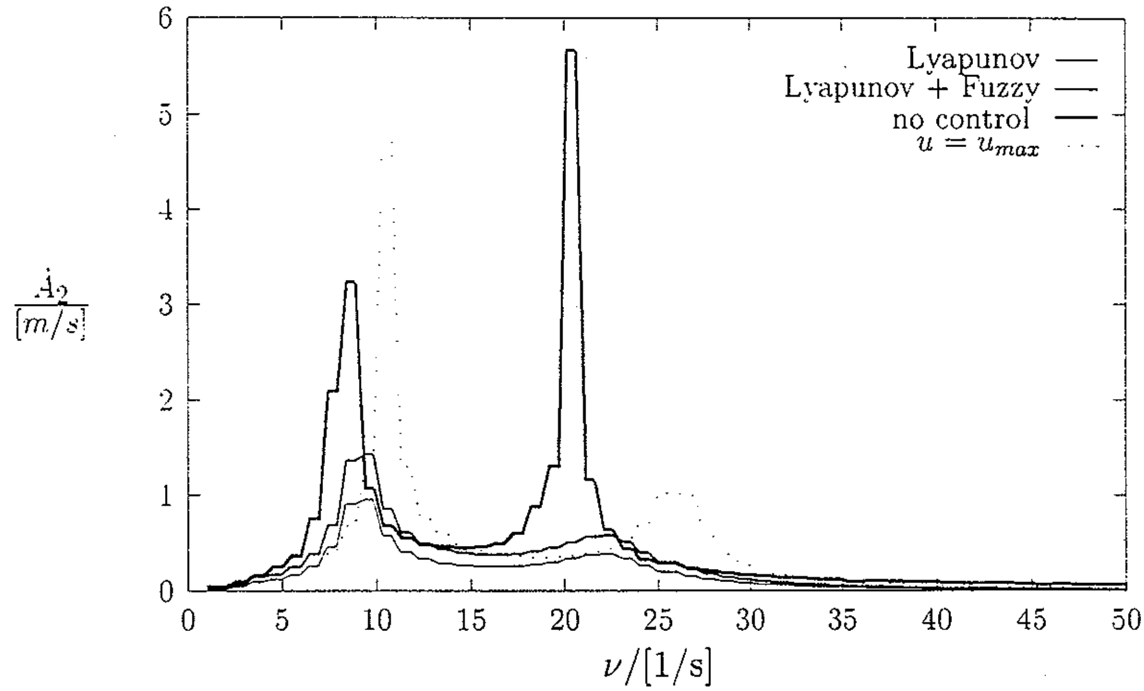

Figure 5.10: Maximum amplitude $\dot{A}_2$ of $\dot{x}_2$ versus excitation frequency ν.

ranges, and these are exactly the frequency domains where suppression in most cases is really wanted. Furthermore, it is important to mention that any time delay during feedback control response may be modelled by an additional uncertainty in all measured state space variables. It should be also mentioned at this point that a possible chattering effect along Π_j will be suppressed by adding the Fuzzy controller near the switching surface.

5.5 References

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